Variables and Patterns

Focus on Algebra

Name: ______________________ Hour: _____

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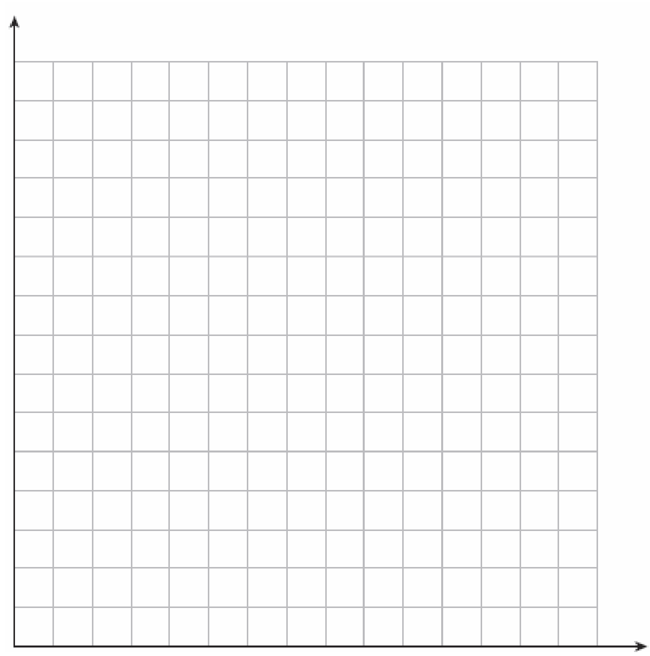
Investigation 1.1

Consider all of the rectangles with a perimeter of 20 units. Describe the relationship between the lengths and the areas of these rectangles.

How does this relationship show up in a table?

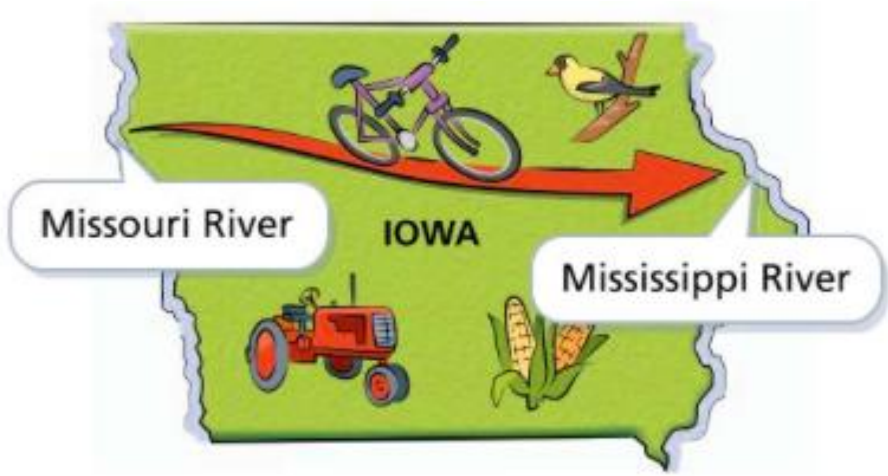
Length									
Width									
Area									

How does this relationship show up in a graph?



The bicycle was invented in 1791. Today, people around the world use bicycles for daily transportation and recreation. Many spend their vacations taking organized bicycle tours.

For example, the RAGBRAI, which stands for Register's Annual Great Bicycle Ride Across Iowa, is a weeklong cycling tour across the state of Iowa. Cyclists start by dipping their back bicycle wheels into the Missouri River along Iowa's western border. They end by dipping their front wheels into the Mississippi River on Iowa's eastern border.



Sidney, Celia, Liz, Malcolm, and Theo heard about the RAGBRAI. The five college students decide to operate bicycle tours as a summer business. They choose a route along the ocean from Atlantic City, New Jersey, to Colonial Williamsburg, Virginia. The students name their new business Ocean Bike Tours.

The Ocean Bike Tours business partners think their customers could ride between 60 and 90 miles a day. Using that guideline, a map, and campground information, they plan a three-day tour route. The business partners also plan for rest stops and visits to interesting places. To finalize plans, they need to answer one more question:

- How are the cyclists' speed and distance likely to change throughout a day?

An answer to that question could only come from a test ride. Because this is difficult to do in school, you can get some ideas by doing a jumping jack experiment. This experiment will test your own physical fitness.

In this experiment, there are two quantities involved, the number of jumping jacks and time. The number of jumping jacks changes over time.

Suppose you did jumping jacks as fast as possible for a 2-minute test period.

- How many jumping jacks do you think you could complete in 2 minutes?

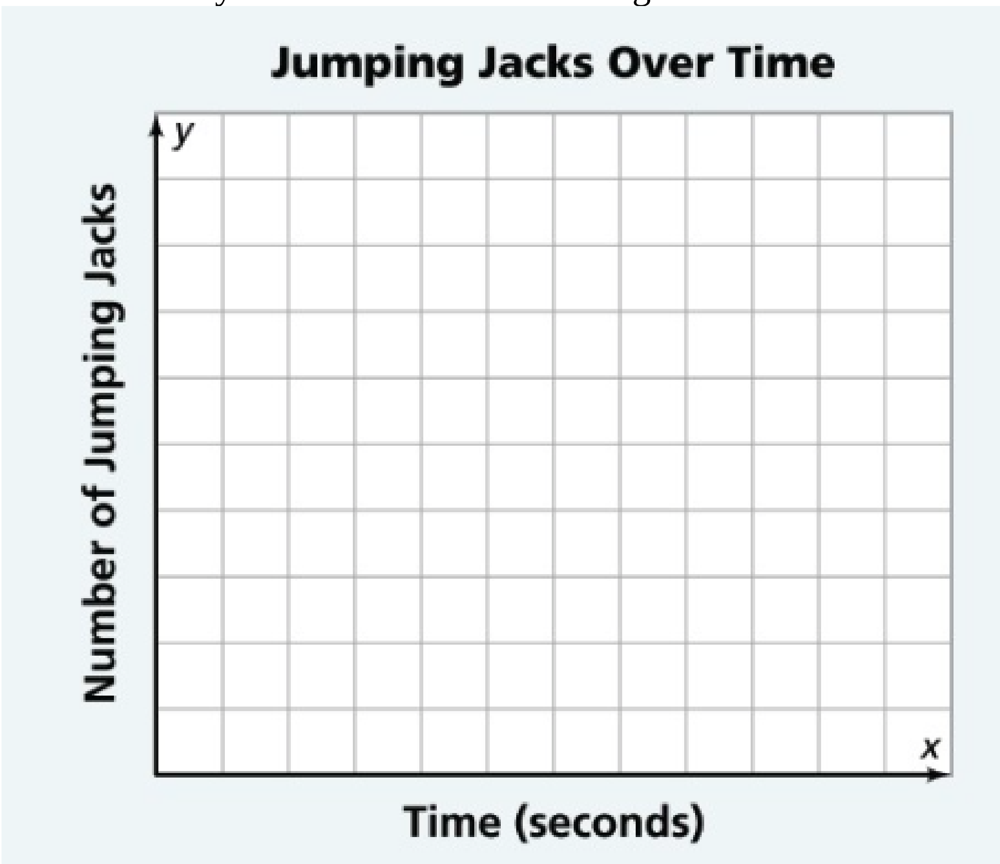
- How do you think your jumping jack rate would change over the 2-minute test?

A. Do the jumping jack fitness test with help from a timer, a counter, and a recorder. Enter the total number of jumping jacks after every 10 seconds in a data table:

Jumping Jack Experiment

Time (seconds)	**0**	**10**	**20**										
Total Number of Jumping Jacks													

B. Record your data on the coordinate grid shown below.



C. How did the jumping jack rate (number per second) change over time?

   1. How is the change over time shown in the data table?

   2. How is the change over time shown in the graph?

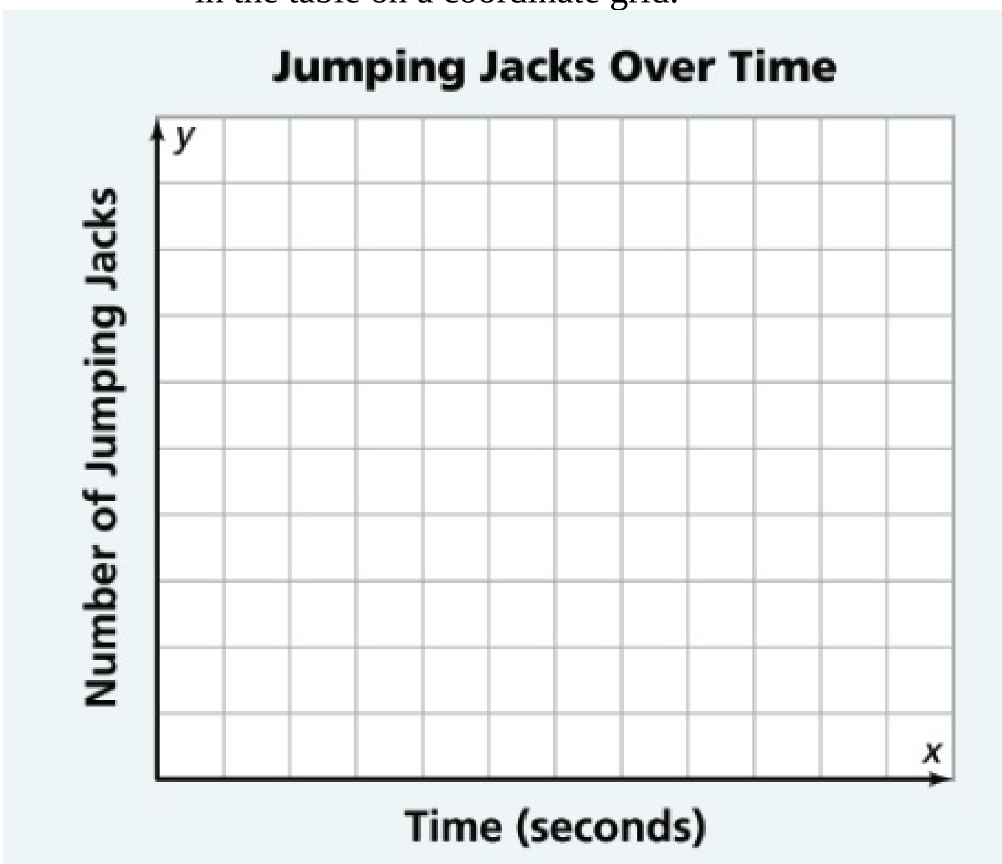
D. Use your jumping jack data. What can you say about the cyclists' speed during the Ocean Bike Tours ride?

E. One group said, "Our jumper did 8 jumping jacks for every 10 seconds."

1. a. Complete the table to show results if a student jumped at a steady pace matching that ratio over 60 seconds.

Time (seconds)	0	10	15	20		30		40		50		60
Total Number of Jumping Jacks		8	12		20		28		36		44	

b. Plot the points corresponding to the (*time, jumping jack total*) pairs in the table on a coordinate grid.



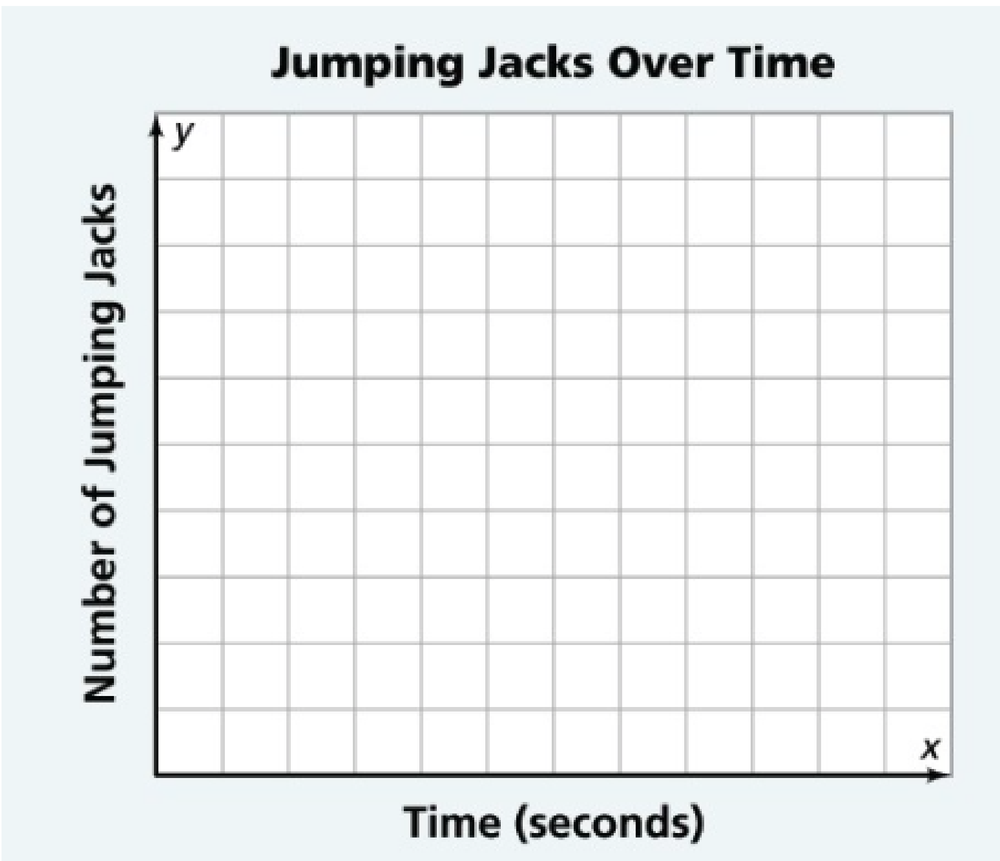
Describe the pattern you see.

2. **a.** Another group's jumper did 4 jumping jacks for every 6 seconds. Complete the table to show results if a student jumped at a steady pace matching that ratio over 30 seconds.

Jumping Jack Experiment

Time (seconds)	0	6	9	12			30
Total Number of Jumping Jacks		4			10	12	

b. Plot the points corresponding to the (*time*, *jumping jack total*) pairs in the table on the coordinate grid. Describe the pattern you see.



Compare the table and graph patterns in parts (1) and (2).

1.1 Summary

Focus Question: How can you construct a graph from a table of data that depicts change over time?

How is this pattern of change represented in the graph?

What is a variable?

Investigation 1.2

In the jumping jack experiment, the number of jumping jacks and time are variables. A **variable** is a quantity that may take on different values. One way in which values of real-life variables may change is with the passage of time. You saw this in the jumping jack experiment. The number of jumping jacks changes based on the elapsed time.

How were the number of jumping jacks and time related?

How was this pattern represented in the table and graph?

The jumping jack experiment gives some ideas about what cyclists might expect on a daylong trip. To be more confident, the Ocean Bike Tours business partners decide to test their bike tour route.

The cyclists begin their bike tour in Atlantic City, New Jersey, and ride south to Cape May.

Sidney follows the cyclists in a van with a trailer for camping gear and bicycles. Every half-hour, he records in a table the distances the cyclists have traveled from Atlantic City.

Atlantic City to Cape May

Time (h)	Distance (mi)
0	0
0.5	8
1.0	15
1.5	19
2.0	25
2.5	27
3.0	34
3.5	31
4.0	38
4.5	40
5.0	45

- As time increases, how does the distance change?

From Cape May, the cyclists and the van take a ferry across Delaware Bay to Lewes (LOO-is), Delaware. They camp that night in a state park along the ocean.

The business partners examine Sidney's (*time*, *distance*) data. They hope to find patterns that might help them improve the Ocean Bike Tours route and schedule. First, they have to answer this question:

- What story does the pattern in the table tell?

A.

1. Plot the (*time*, *distance*) data pairs on the coordinate grid.

Distance Traveled Over Time

Distance (mi)

Time (h)

2. What interesting patterns do you see in the (*time*, *distance*) data?

3. Explain how the patterns are shown in the table.

4. Explain how the patterns are shown on the graph.

B.

1. At what times in the trip were the cyclists traveling fastest? At what times were they traveling slowest?

   Fastest:

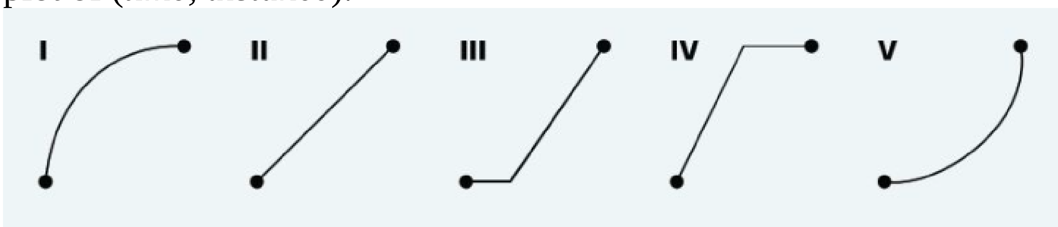
   Slowest:

2. Explain how your answer is shown in the table.

3. Explain how your answer is shown by the pattern of points on the graph.

C. Connecting the points on a graph can help you see patterns more clearly. It also helps you consider what is happening in the intervals between the points. Different ways of connecting the given data points tell different stories about what happens between the points.

Consider the data (4.5, 40) and (5.0, 45) from the first day of the Ocean Bike Tours trip. Here are five different ways to connect the graph points on the plot of (*time*, *distance*).



Match the given connecting paths to these travel stories.

______ **1.** Celia rode slowly at first and gradually increased her speed.

______ **2.** Theo rode quickly and reached the Cape May ferry dock early.

______ **3.** Malcolm had to fix a flat tire, so he started after the others.

______ **4.** Tony and Sarah started off fast. They soon felt tired and slowed down.

______ **5.** Liz pedaled at a steady pace throughout this part of the trip.

D. What are the advantages and disadvantages of tables or graphs to represent a pattern of change?

1.2 Summary

Focus Question: What are the advantages and disadvantages of tables and graphs in representing and describing the pattern of change in a variable over time?

Advantages:

Disadvantages:

Investigation 1.3

From Lewes to Chincoteague Island

Stories, Tables, and Graphs

On the second day of the bike tour test run, the team leaves Lewes, Delaware, and rides through Ocean City, Maryland. The team stops on Chincoteague (SHING kuh teeg) Island, Virginia. Chincoteague Island is famous for its annual pony auction. Here, the team camps for the night.

Did You Know?

Assateague (A suh teeg) Island is home to herds of wild ponies. The island has a harsh environment of ocean beaches, sand dunes, and marshes. To survive, these sturdy ponies eat salt marsh grasses, seaweed, and even poison ivy.

To keep the population of ponies under control, an auction is held every summer. During the famous "Pony Swim," the ponies for sale swim across a quarter mile of water to Chincoteague Island.

Malcolm and Liz drove the tour van on the way from Lewes to Chincoteague. They forgot to record time and distance data. Fortunately, they wrote some notes about the trip.

Entry 1: We started at 8:00 A.M. and rode against a strong wind until our midmorning break. Entry 2: About midmorning, the wind shifted to our backs. Entry 3: Around noon, we stopped for BBQ lunch and rested for about an hour. By this time we had traveled about halfway to Chincoteague.	Entry 4: Around 2:00 P.M., we stopped for a brief swim in the ocean. Entry 5: At about 4:00 P.M., all of the riders were tired. There were no bike lanes. So we packed the bikes in the trailer and rode in the van to our campsite in Chincoteague. We took 9 hours to complete today's 80-mile trip.

A. Make a table of (*time*, *distance*) values to match the story told in Malcolm and Liz's notes.

Lewes to Chincoteague

Time									
Distance									

B. Sketch a coordinate graph that shows the information in the table.

Distance Traveled Over Time

Distance (mi)

Time (h)

Does it make sense to connect the points on the graph? **Explain** your reasoning.

C. Explain how the entries in your table and graph illustrate the trip notes.

D. Which representation of the data (table, graph, or written notes) best shows the pattern of change in distance over time**? Explain.**

1.3 Summary

Focus Question: Which representation of data – table, graph, or written notes – seems to better show patterns of change in distance over time, and why?

What was their average speed over the day's trip?

What would the graph have looked like if the cyclists had traveled at this average speed all day?

Distance Traveled Over Time

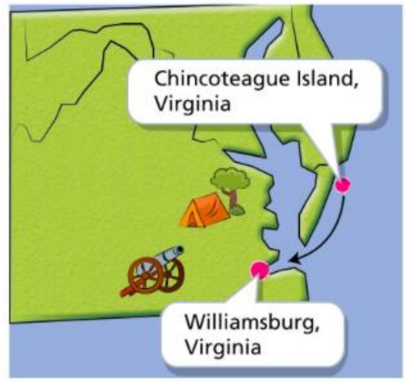
Distance (mi)

Time (h)

Investigation 1.4

From Chincoteague to Colonial Williamsburg

Average Speed



Did You Know?

Williamsburg was the political, cultural, and educational center of Virginia from 1699 to 1780. Virginia was the largest, most populous, and most influential of the American colonies.

Near the end of the Revolutionary War, the capital of Virginia was moved to Richmond. For nearly 150 years afterward, Williamsburg was a quiet town.

Then, in 1926, a movement began to restore and preserve the city's historic buildings. Today, Williamsburg is a very popular tourist destination.

Malcolm noticed that, on Day 1, the cyclists sometimes went very fast or very slow in any given hour. He also noticed that the cyclists covered 45 miles in 5 hours.

Atlantic City to Cape May

Time (h)	0	0.5	1.0	1.5	2.0	2.5	3.0	3.5	4.0	4.5	5.0
Distance (mi)	0	8	15	19	25	27	34	31	38	40	45

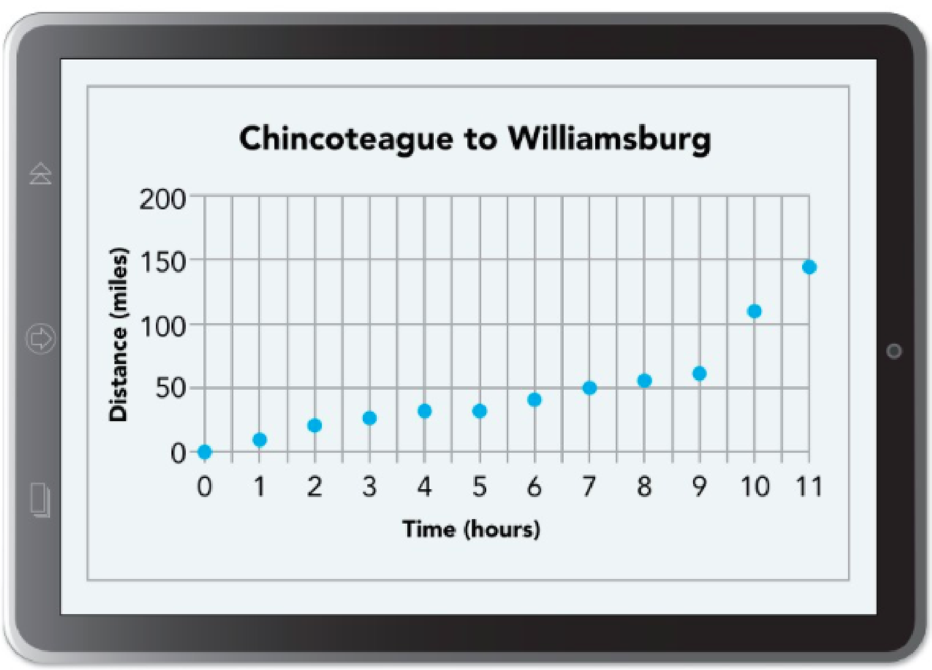
- Malcolm claims that, on average, the cyclists covered 9 miles per hour. Is he correct?

- Did the cyclists actually cover 9 miles per hour in any one hour on Day 1? Explain.

The **average speed** per day is the rate in miles per hour for that day. Malcolm was curious to know what the average speed for Day 3 would be.

On the third day of the bike tour test run, the team travels from its campsite on Chincoteague Island to Williamsburg, Virginia. Here, they visit the restored colonial capital city.

Malcolm drove and Sarah rode in the tour van on the way from Chincoteague to Williamsburg. They made a graph showing the cyclists' progress each hour.



A. Make a table of the (*time*, *distance*) value pairs shown in the graph.

Time											
Distance											

1. What does the point with coordinates (3, 25) tell about the cyclists' progress?

2. Which points on the graph have coordinates (9, 60) and (10, 110)? What do those coordinates tell about the cyclists' time, distance, and speed on Day 3?

3. What was the cyclists' average speed in miles per hour for the trip? How can you find this from the graph? From the table?

B. The team has to cross the Chesapeake Bay Bridge and Tunnel. Then, they travel on an interstate highway from Norfolk to Williamsburg. So, the team bikes for only the first part of the trip.

1. Based on the graph and your table, when did the team put its bikes on the trailer and begin riding in the van?

2. What was the team's average speed for the trip time completed on bikes?

3. What was the team's average speed for the trip time completed in the van?

4. How are differences in travel speed shown in the graph?

C. A very strong cyclist makes the trip from Chincoteague to Williamsburg in 8 hours pedaling at a constant speed.

1. At what speed did the cyclist travel?

2. Describe the graph of (*time, distance*) data for the trip.

1.4 Summary

Focus Question: How do you calculate average speed for a trip? How do a table and graph of (*time, distance*) data show speed?

- How can you tell from the graph where the fastest speed is occurring?

Investigation 1 Reflection

The Problems in this Investigation helped you to think about variables and patterns relating values of variables. In particular, they helped you develop understanding and skill in the use of data tables and graphs in order to study quantities or variables that change over time.

This Investigation challenged you to use those mathematical tools to find important patterns in the relationships between distance, time, and the speed of moving objects.

You can show patterns of change over time with tables, graphs, and written reports.

1. What are the advantages and disadvantages of showing patterns with tables?

2. What are the advantages and disadvantages of showing patterns with graphs?

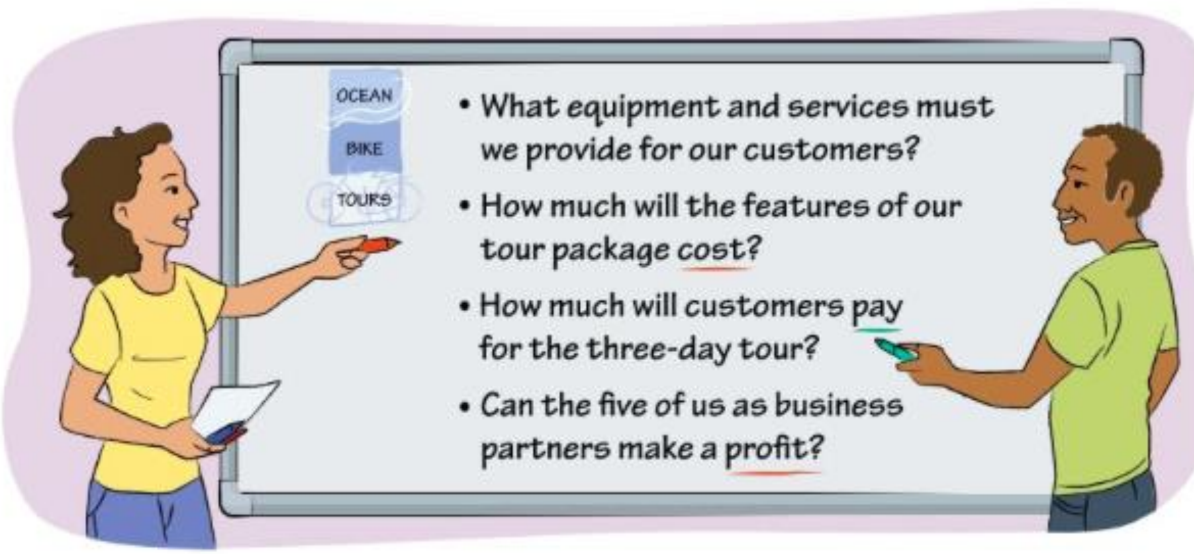
3. What are the advantages and disadvantages of showing patterns with written reports?

4. How do you see patterns in the speed of a moving object by studying (*time*, *distance*) data in tables?

5. How do you see patterns in the speed of a moving object by studying (*time*, *distance*) data in coordinate graphs?

Investigation 2

The test run by the Ocean Bike Tours partners raised many questions.



To make their choices, the five partners decided to do some research. In this Investigation you will use tables, graphs, and words to analyze information from their research and advise the tour business partners.

With your group, make a list of the things the tour operators will have to provide for their customers during the trip. Estimate the cost of each item.

-
-
-
-
-

How much do you think the customers would be willing to pay for the tour?

Based on how much you think the items will cost and what the customers would pay, would the operators of the tour earn a profit?

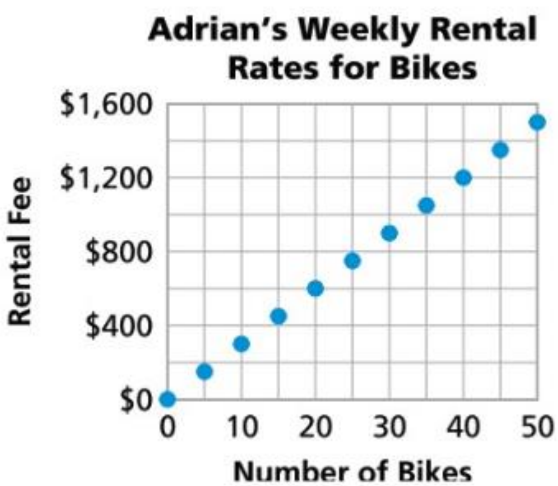
Investigation 2.1

The tour operators decide to rent bicycles for their customers. They get information from two bike shops. Rocky's Cycle Center sends a table of rental fees for bikes.

Bike Rental at Rocky's

Number of Bikes	5	10	15	20	25	30	35	40	45	50
Rental Cost ($)	400	535	655	770	875	975	1,070	1,140	1,180	1,200

Adrian's Bike Shop sends a graph of their rental prices. The number of bikes rented is called the **independent variable**. The rental cost is called the **dependent variable** because the rental cost *depends* on the number of bikes rented.



Graphs usually have the independent variable on the x-axis and the dependent variable on the y-axis.

The Ocean Bike Tour partners need to choose a bike rental shop. Suppose that they ask for your advice.

- Which shop would you recommend?
- How would you justify your choice?

Use entries in the table and the graph to answer the following comparison questions.

A. What are the costs of renting from Rocky and Adrian if the tour needs:
20 bikes?

Rocky: Adrian:

40 bikes?

Rocky: Adrian:

32 bikes?

Rocky: Adrian:

B. About how many bikes can be rented from Rocky or Adrian in the following cases?

1. A group has $900 to spend.

2. A group has $400 to spend.

C. You want to see how rental cost is related to number of bikes.

1. What pattern do you see in the table from Rocky's Cycle Center?

2. What pattern do you see in the graph from Adrian's Bike Shop?

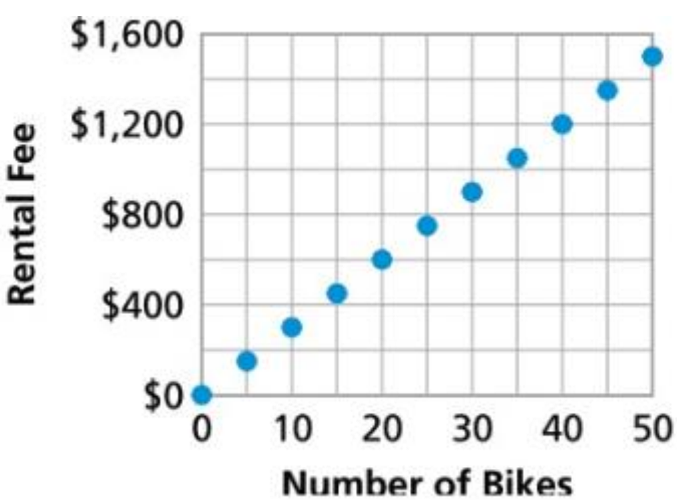
D. How can you predict rental costs for numbers of bikes that are not shown by entries in the table of points on the graph?

E. What information about bike rental costs was easier to get from the table and what from the graph?

F. Which data format is most useful?

2.1 Summary

Focus Question: How do you analyze and compare the relationship between variables given in different representations?



How can you decide which bike shop to choose?

Investigation 2.2

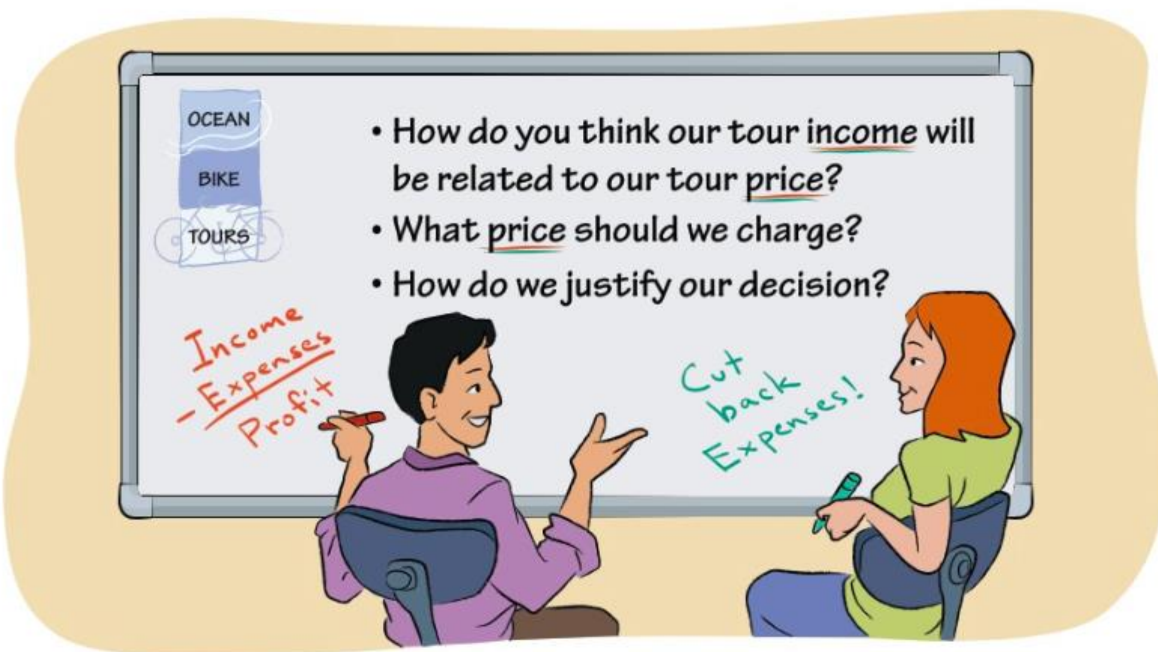
The tour operators have planned a route and chosen a bike rental shop. The next task is to figure out a price to charge for the tour. They want the price low enough to attract customers. They also want it high enough to have **income** that is greater than their expenses. That way their business makes a **profit**.

The partners conduct a survey to help set the price. They ask people who have taken other bicycle tours what they would pay for the planned bike tour.

Prices That Customers Would Pay

Tour Price	$100	$150	$200	$250	$300	$350	$400	$450	$500
Number of Customers	40	35	30	25	20	15	10	5	0

Look carefully at the data relating price and number of customers.



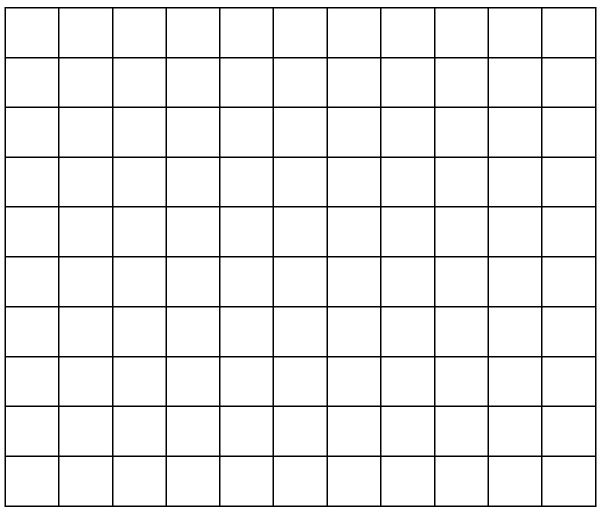
The following questions can help you choose a tour price.

A.

1. Make a graph of the data relating price and number of customers. Which is the independent variable? Which is the dependent variable? Explain how you know.

Independent variable:

Dependent variable:



2. How does the number of customers change as the price increases?

3. How is the change in number of customers shown in the table? How is the change shown by the graph?

4. How would you estimate the number of customers for a price of $175?

For a price of $325?

B.

1. The partners need to know what income to expect from the tour. They extend the (*price, customers*) table as shown below. Complete the table to find how income would be related to price and number of customers.

Tour Price	$100	$150	$200	$250	$300	$350	$400	$450	$500
Number of Customers	40	35	30	25	20	15	10	5	0
Tour Income	$4,000								

2. Make a graph of the (*price*, *income*) data.

3. Describe the pattern relating tour income to tour price. Use a sentence that begins, "As tour price increases, tour income…." Explain **why** that pattern does or does not make sense.

What price should the tour operators charge? Explain why.

2.2 Summary

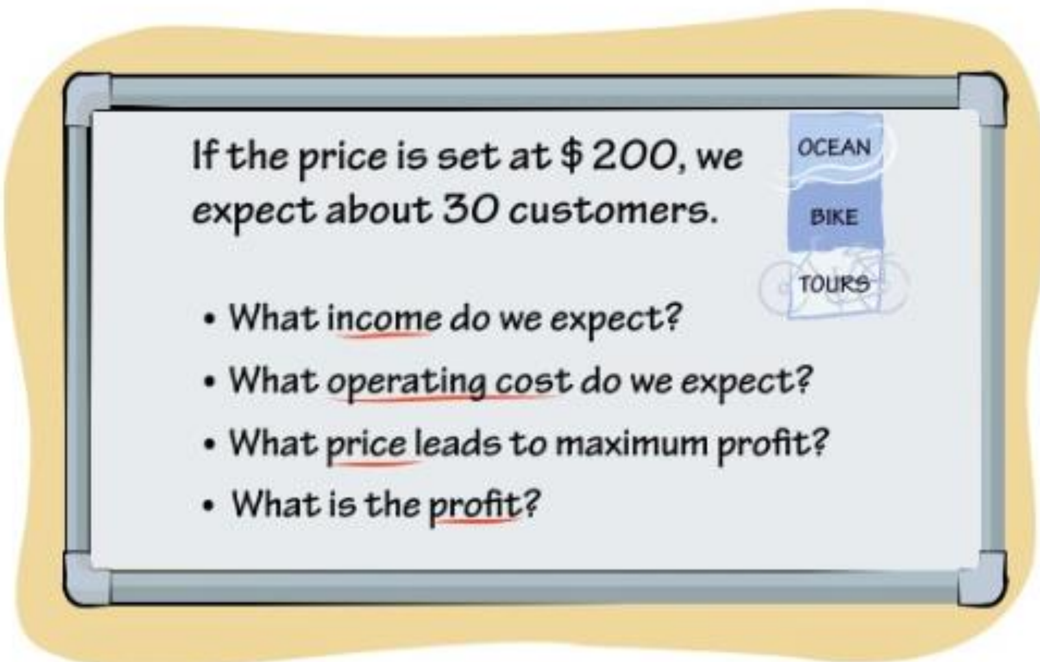
Focus Question: How are the relationships between independent and dependent variables in this Problem different from those in Problem 2.1?

How are the differences shown in tables and graphs of data?

Investigation 2.3

The survey conducted by Ocean Bike Tours showed that income depends on the tour price. The partners want to see if they can make any profit from their business. As well as income, they have to consider the costs of operating the tour. Their research shows that bike rental, camping fees, and food will cost $150 per customer.

The partners want to make a profit. They need to figure out how profit depends on the tour price.



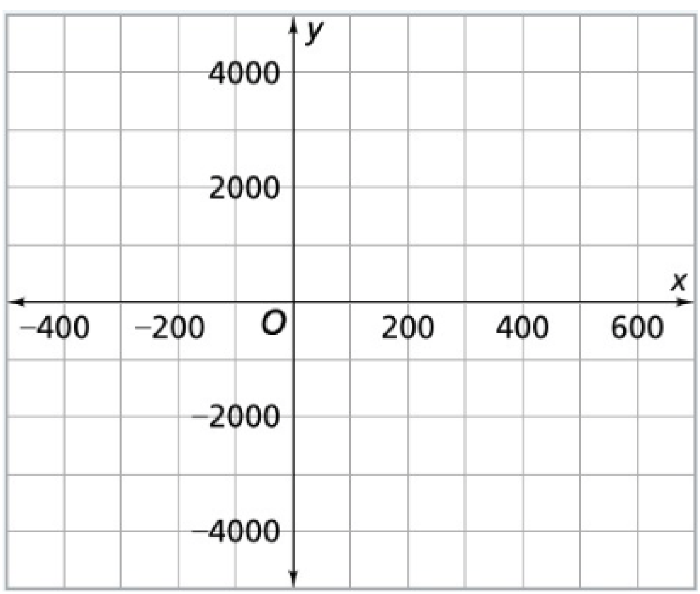
A.

1. The table below shows the relationship between profit and price. Complete the table.

Tour Price	$100	$150	$200	$250	$300	$350	$400	$450	$500
Number of Customers	40	35	30	25	20	15	10	5	0
Tour Income ($)	4,000								
Operating Cost ($)	6,000								
Tour Profit or Loss ($)	-2,000								

2. Celia and Malcolm want a picture of profit prospects for the tour business. They need to graph the (*price, profit*) data. Some of the data are negative numbers. Those numbers represent possible losses for the tour operation.

   The key to graphing data that are negative numbers is to extend the *x*- and *y*-axis number lines. Both the *x*- and *y*-axes can be extended in the negative direction. This gives a grid like the one shown below. Use the grid to sketch a graph for the (*price, profit*) data points from the table in part (1).



3.

a. Describe the pattern in the table in part (1) and the graph in part (2)

b. Explain why the pattern occurs.

c. Think about the analysis of profit predictions. What tour price would you suggest? Explain your reasoning.

B. In January, the partners thought about offering a winter bike tour. They looked at the forecast for the next four days. They wrote down the number of degrees above or below each day's average temperature.

Degrees Above or Below Average Temperature

x	0	1	2	3	4
y	–1	5	–3	–5	2

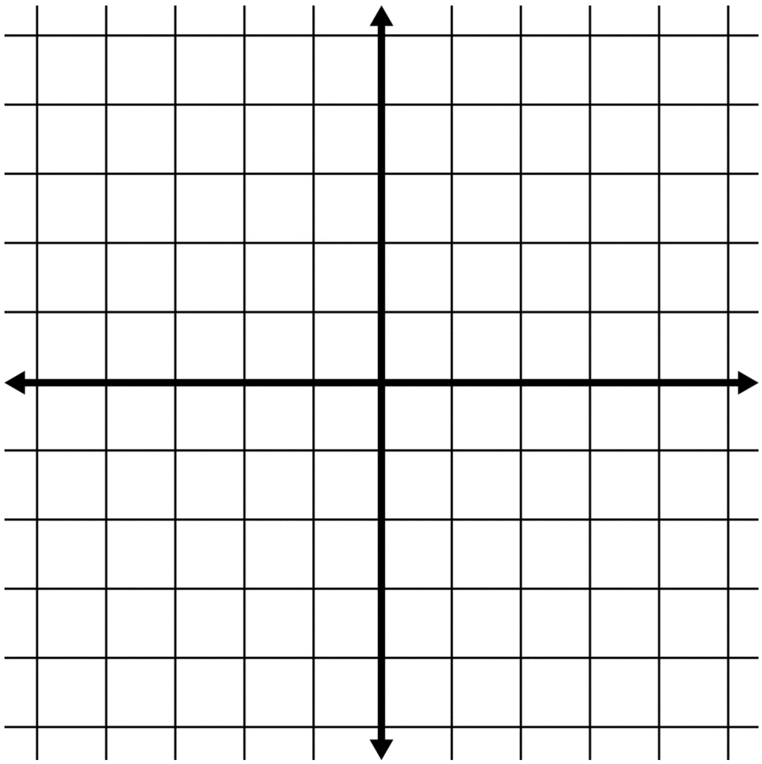
They did not see any pattern, so they checked the temperatures for the previous five days. They compared those temperatures to the average. They recorded their data for all nine days in the table below.

Degrees Above or Below Average Temperature

x	–4	–3	–2	–1	0	1	2	3	4
y	–2	4	–3	1	–1	5	–3	–5	2

1. What do the *x*- and *y*-values represent?

2. Plot the pairs of (*x*, *y*) values in the table on a coordinate grid. Label each point with its coordinates.



3. Describe the pattern of change that relates the two variables.

C.

1. Suppose that you are standing at the point with coordinates (3, 4). Tell how you would move on the grid lines to reach the points below.

a. (-3, 4)

b. (-3, -4)

c. (3, -4)

d. (1.5, -2)

2. How far would you have to move on the grid lines to travel between each pair of points?
   a. (3, 4) to (-3, 4)

   b. (3, 4) to (3, -4)

   c. (3, 4) to (-3, -4)

D.

1. Jakayla was looking at the points (3, 4), (-3, 4), (-3, -4), and (3, -4). She said that the locations of the points with different signs are mirror images of each other. Does Jakayla's conjecture make sense? Explain.

2. Mitch says this is like a reflection. Does Mitch's comment make sense?

2.3 Summary

Focus Question: How are the variables *tour income* and *tour profit* related to each other?

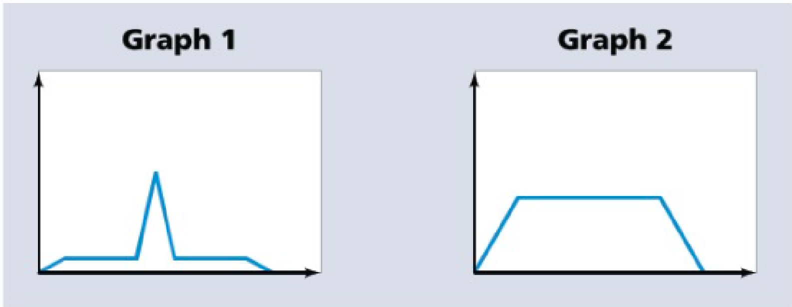
How do you plot data points with one or both coordinates negative?

Investigation 2.4

Information about variables is often given by coordinate graphs. So, it is important to be good at reading the "story" in a graph. Here are some questions to ask when you look at a graph.

- What are the variables?
- Do the values of one variable seem to depend on the values of the other?
- What does the shape of the graph say about the relationship between the variables?

For example, the number of cars in your school's parking lot changes as time passes during a typical school day. Graph 1 and Graph 2 show two possibilities for the way the number of parked cars might change over time.



- Describe the story each graph tells about the school parking lot.
  Graph 1:

  Graph 2:

- Do either of these graphs show the pattern that happens at MacDonald Middle School? If not, draw a new graph.

- How could you label the graph you chose so that someone else would know what it represents?

Questions A – H describe pairs of related variables. For each pair, do the following:

- Decide what the variables are.
- Decide which variable is the dependent variable and which is the independent variable.
- Think about what a graph or table of these data would look like.
- Find the graph at the end of the Problem that tells the story of how the variables are related. *If no graph fits the relationship as you understand it, sketch a graph of your own.*
- Explain what the graph tells about the relationship of the variables.
- Give the graph a title.

A. The number of students who go on a school trip is related to the price of the trip for each student.

Independent variable:
Dependent variable:
Graph:

Explanation:

Title:

B. When a skateboard rider goes down one side of a half-pipe ramp and up the other side, her speed changes as time passes.

Independent variable:
Dependent variable:
Graph:

Explanation:

Title:

C. The water level changes over time when someone fills a tub, takes a bath, and empties the tub.

Independent variable:
Dependent variable:
Graph:

Explanation:

Title:

D. The waiting time for a popular ride at an amusement park is related to the number of people in the park.

Independent variable:
Dependent variable:
Graph:

Explanation:

Title:

E. The daily profit or loss of an amusement park depends on the number of paying customers.

Independent variable:
Dependent variable:
Graph:

Explanation:

Title:

F. The number of hours of daylight changes over time as the seasons change.

Independent variable:
Dependent variable:
Graph:

Explanation:

Title:

G. The daily profit or loss of an outdoor skating rink depends on the daytime high temperature.

Independent variable:
Dependent variable:
Graph:

Explanation:

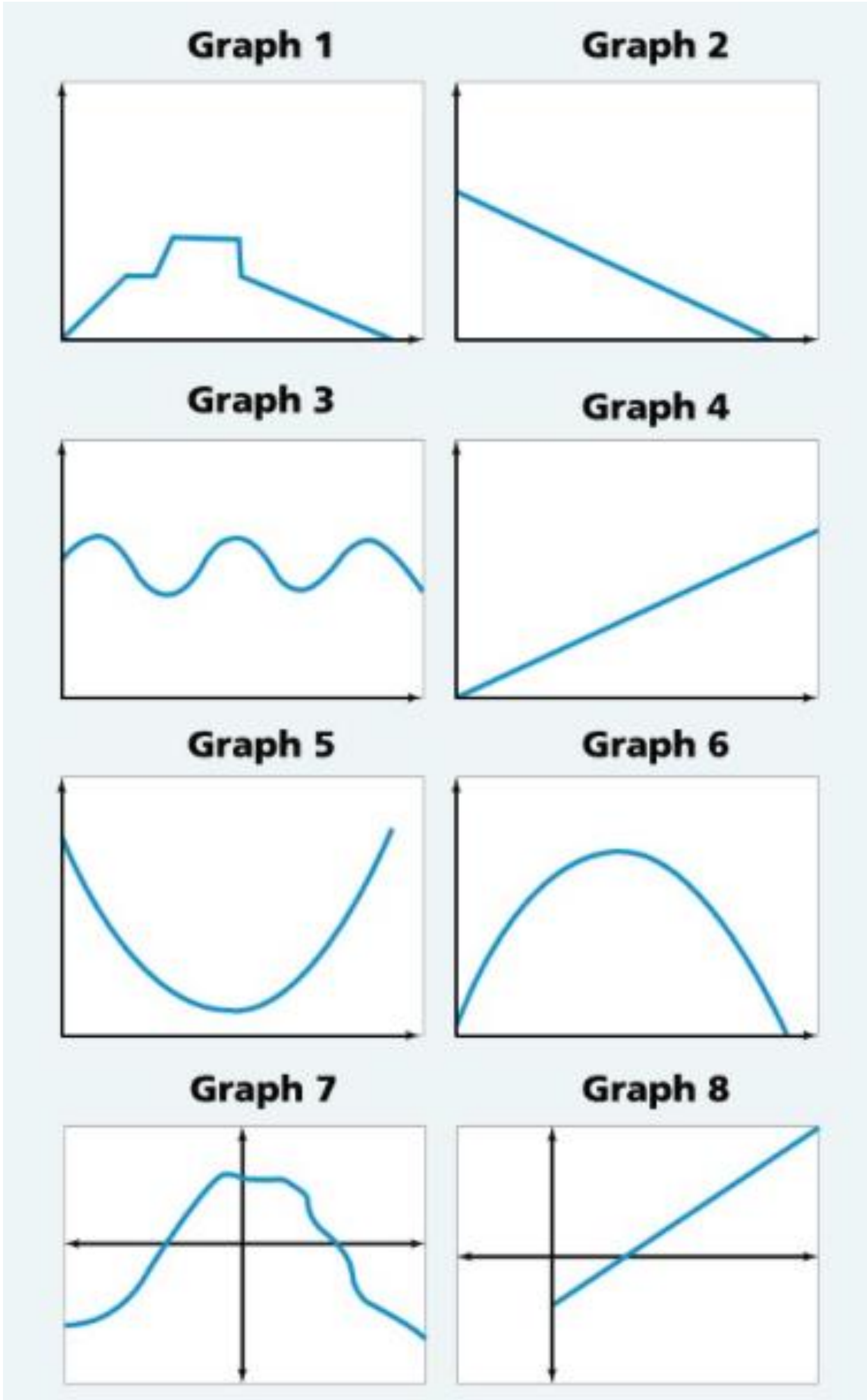
Title:

H. Weekly attendance at a popular movie changes as time passes from the date the movie first appears in theaters.

Independent variable:
Dependent variable:
Graph:

Explanation:

Title:


Graph 1
Graph 2
Graph 3
Graph 4
Graph 5
Graph 6
Graph 7
Graph 8

2.4 Summary

Focus Question: When the relationship between dependent and independent variables is displayed in a graph, what can you learn about the relationship from a rising graph, a level graph, and a falling graph?

Sketch a graph of a dog's weight over time:

Investigation 2 Reflection

In this Investigation, you looked at patterns relating the values of variables. You also thought about the ways that those patterns are shown in tables of values and coordinate graphs. The following questions will help you to summarize what you have learned.

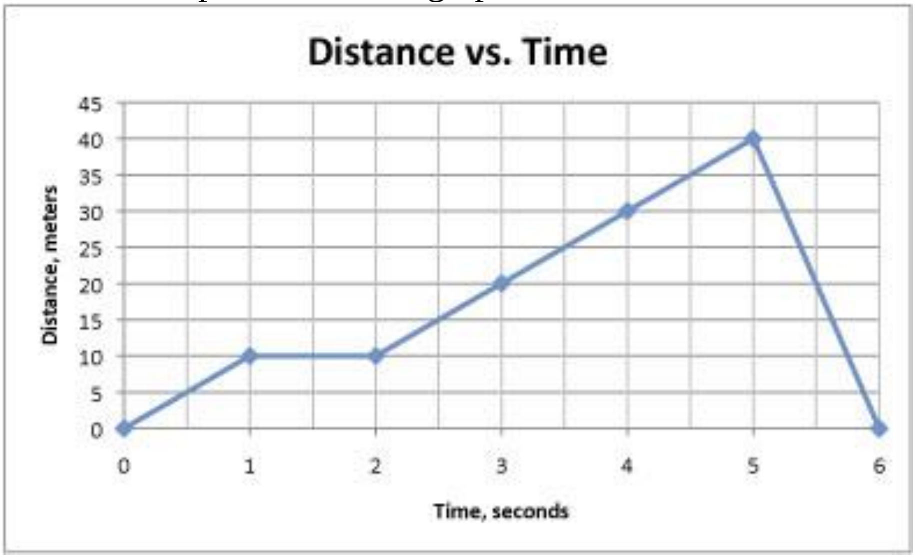
1. The word *variable* is used often to describe conditions in science and business.
   - **a.** Explain what the word *variable* means when it is used in situations like those you studied in this investigation.
   - **b.** When are the words *independent* and *dependent* used to describe related variables? How are they used?
2. Suppose the values of a dependent variable increase as the values of a related independent variable increase. How is the relationship of the variables shown in each of the following?
   - **a.** a table of values for the two variables
   - **b.** a graph of values for the two variables

3. Suppose the values of a dependent variable decrease as the values of a related independent variable increase. How is the relationship of the variables shown in each of the following?

a. a table of values for the two variables

b. a graph of values for the two variables

4. Describe the patterns in this graph.



To prepare for the quiz:

- Know how to write a pattern sentence. You have many examples in your notes!
- Know how to turn written notes into a table and graph. Revisit Lesson 1.3 from Investigation 1.
- Know how to identify independent and dependent variables and be able to explain your choice.
- Know how to create a title for a table and a graph.

Investigation 3.1

On the last day of the Ocean Bike Tours trip, the riders will be near Wild World Amusement Park. They want to plan a stop there.

What variables would affect the cost of the amusement park trip?

How would those variables affect the cost?

Malcolm finds out that it costs $21 per person to visit Wild World. Liz suggests that they make a table or graph relating admission price to the number of people. However, Malcolm says there is a simple rule for calculating the cost:

The cost in dollars is equal to 21 times the number of people.

He wrote the rule as the statement:

cost = 21 x *number of people*

Liz shortens Malcolm's statement by using single letters to stand for the variables. She uses c to stand for the cost and n to stand for the number of people:

$$c = 21 \times n$$

When you multiply a number by a letter variable, you can leave out the multiplication sign. So 21n means 21 x n. You can shorten the statement even more:

$$c = 21n$$

So, 21n is an **expression** for the total cost C. You obtain the total cost by multiplying 21, the cost per person, by n, the number of people. The fact that C and 21*n* are equal gives the **equation** C=21*n*. Here, the number 21 is called the **coefficient** of the variable *n*.

The equation c=21n involves one calculation. You multiply the number of customers *n* by the cost per customer, $21. Many common equations involve one calculation.

A. Theo wants to attract customers for the bike tour. He suggests a discount of $50 off the regular price for early registration.

1. a. What is the discounted price if the regular tour price is $400?

b. $500?

c. $650?

2. Write an equation that represents the relationship of discounted price *D* to regular tour price *P*.

B. When the Ocean Bike Tours partners set a price for customers, they need to find the 6% sales tax.

1. a. What is the sales tax if the tour price is $400?

b. What is the sales tax if the tour price is $500?

c. What is the sales tax if the tour price is $650?

2. Write an equation that represents the relationship of the amount of sales tax T to tour price P.

C. Suppose a professional cyclist sustained a speed of about 20 miles per hour over a long race.

1. a. About how far would the cyclist travel in 2 hours?

b. 3 hours?

c. 3.5 hours?

2. At a speed of 20 miles per hour, how is the distance traveled d related to the time t (in hours)? Write an equation to represent the relationship.

3. Explain what information the **coefficient** of t represents.

D. The trip from Williamsburg, Virginia, to Atlantic City, New Jersey, is about 350 miles.

1 **a**. How long will the trip take if the average speed of the van is 40 miles per hour?

b. 50 miles per hour?

c. 60 miles per hour?

2. Write an equation that shows how total trip time *t* depends on average driving speed *s*.

3.1 Summary

Focus Question: In what kinds of situations will the equation for the relationship between dependent and independent variables be in the form of $= x + k,\ y = x - k,\ y = \frac{x}{k}$?

Investigation 3.2

There are many relationships between variables that you can write as algebraic equations. One simple type is especially important.

Relationship:	cost of admissions to number of customers	sales tax to Ocean Bike Tours price	distance to time traveled by cyclist
Equation:	$c = 21n$	$T = 0.06P$	$d = 20t$
Description of Coefficient:	price per customer	tax rate per dollar	average speed in miles per hour

Relationships with rules in the form $y = mx$ occur often. It is important to understand the patterns in tables and graphs that those relationships produce. It is also useful to understand the special information provided in each case by m – the coefficient of x.

In these equations, the coefficient tells the **rate of change** in the dependent variable as the independent variable increases steadily.

How is the rate of change represented in an equation, table, and graph?

The questions in this Problem will develop your understanding and skill in working with rates in many different situations.

A. When the bike tour is over, the riders will put their bikes and gear into vans and head back to Atlantic City.

1. Complete the rate table to show how distance depends on time for different average speeds.

Time (h)	Distance for Speed of 50 miles/hour	Distance for Speed of 55 miles/hour	Distance for Speed of 60 miles/hour

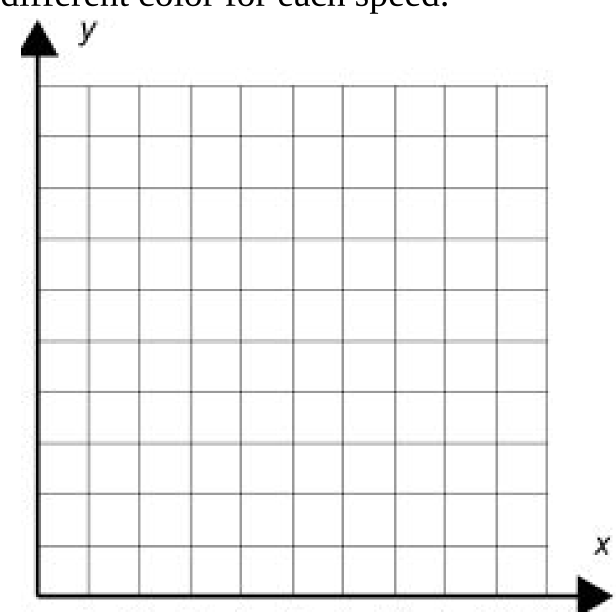
2. Write an equation to show how distance d and time t are related for travel at each speed.

a. 50 miles per hour

b. 55 miles per hour

c. 60 miles per hour

3. Graph the (*time, distance*) data for all three speeds on the same coordinate grid. Use a different color for each speed.



4. For each of the three average speeds:

a. Look for patterns relating distance and time in the table and graph. Explain how the pattern shows up in the table and graph.

b. Theo observed that the coefficient of the independent variable in each equation is the average speed or unit rate. Is he correct? Explain.

5. **a**. Look how you can use the table, graph, or equation to find the distance when t = 6 hours.

b. How can you use the table, graph, or equation to find the time when the distance is 275 miles? Explain.

B. A smartphone plan charges $.03 per text message.

1. **a.** Make a table of monthly charges for 0; 500; 1000, 1,500; 2,000; and 2,500 text messages.

Number of text messages	**Cost**
0	
500	
1,000	
1,500	
2,000	
2,500	

b. Use the table. What is the cost for 1,000 messages?

For 1,725 messages?

c. Use the table. How many text messages were sent in a month if the charge for the messages if $75?

$60?

$18?

2. a. How is the monthly charge *B* for text messages related to the number of text messages *n*?

Write an equation that represents the monthly charge for n messages.

b. Use the equation you wrote in part (a) to find the cost for 1,250 text messages in one month.

3. **a**. Sketch a graph of the relationship between text message charges and number of messages.

b. Explain how you could use the graph to answer the questions in parts (1b), (1c), and (2b).

C. The metric and English system units for measuring length are related. The rule is that 1 inch is equal to about 2.5 centimeters.

1. What is the length in centimeters of a line segment that measures 5 inches?

12 inches?

7.5 inches?

2. How can you calculate the length in centimeters C of an object that you have measured in inches I?

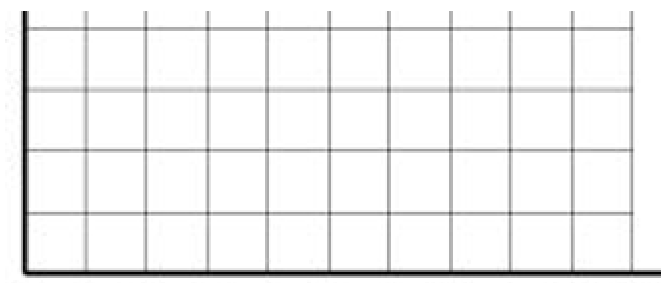
Write an equation to represent this calculation. Use the equation to find the number of centimeters that corresponds to 12 inches.

3. What is the approximate length in inches of a line segment that measures 10 centimeters?

30 centimeters?

100 centimeters?

4. Sketch a graph of the relationship between length in centimeters and length in inches in part (2). Explain how you could use the graph to answer the questions in parts (1) and (3).

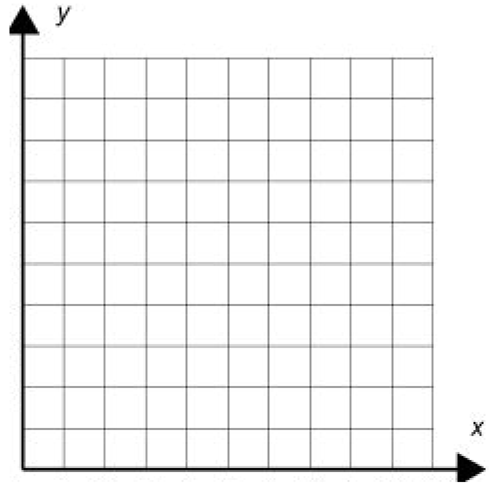


D. The equations you wrote in Questions A-C all have form $y = mx$.

1. Complete the table of (x, y) values below. Use whole-number values of x from 0 to 6.

x	0	1	2	3	4	5	6
$y = 2x$							
$y = 0.5x$							
$y = 1.5$							
$y = x$							

Use your table to make a graph.



2. Explain the connection between the number m and the pattern in the table of values and graph of $y = mx$.

3. **a**. Explain how you can find the value of y using a table, graph, or equation if $x = 2$.

b. Explain how you can find the value of x using a table, graph or equation if $y = 6$.

4. Write a story to represent each equation in part (1).

E. What similarities and differences do you find in the equations, tables, and graphs for the relationships in Questions A-D?

3.2 Summary

Focus Question: What can you tell about the relationship between dependent and independent variables in an equation of the form $y = mx$?

How is that relationship shown in a table and a graph of sample (x,y) values? Why is the point $(1,m)$ on every graph?

Investigation 3.3

Each equation you wrote in Investigation 3.1 and 3.2 involved only one operation (+, - , x, ÷). Some equations involve two or more arithmetic operations. To write such equations, you can reason just as you do with one-operation equations:

- Identify the variables.
- Work out some specific numeric examples. Examine them carefully. Then, look for patterns in the calculations used.
- Write a rule in words to describe the general pattern in the calculations.
- Convert your rule to an equation with letter variables and symbols.
- Think about whether your equation makes sense. Test it for a few values to see if it works.

Liz and Theo want to visit Wild World with their friends. Theo checks if the part offers special prices for groups larger than 3 people. He finds this information on the park's Web site:

A. Study the rule.

1a. Complete the following table:

Number in Group	4	8	12	16	20	24	28	32	36	40
Price ($)										

b. Sketch a graph of the data.

c. Describe the pattern of change that shows up in the table and graph.

2 a. Describe in words how you can calculate the admission price for a group with any number of people.

b. Write an equation relating admission price *p* to group size *n*.

c. How is this pattern of change in prices for group admissions similar to the pattern of change for the equations in Problem 3.2?

How is it different?

3a. Describe how you can use the table, graph, or equation to find the cost for 18 people.

b. Describe how you can use the table or graph to find the number of people in the group if the total charge is $350 or $390.

B. Admission to Wild World includes a bonus card with 100 points that can be spent on rides. Rides cost 6 points each.

1. Complete the table below to show a customer's bonus card balance after various numbers of rides.

Bonus Card Balance

Number of Rides	0	1	2	3	5	7	10	15
Points on Card	100							

2. Explain how you can calculate the number of points left after any number of rides.

3. Write an equation showing the relationship between points left on the bonus card and number of rides taken.

4. How does cost per ride appear in the equation?

How does the number of bonus points at the start appear in the equation?

5. Sketch a graph of the relationship between points left and number of rides for up to 20 rides.

Describe the relationship between the variables.

C. Liz wonders whether they should rent a cart to carry their backpacks. The equation $c = 20 + 5h$ shows the cost in dollars c of renting a cart for h hours.

1. What information does each number and variable in the expression $20 + 5h$ represent?

2. Use the equation to complete the following table.

Golf Cart Rental

Number of Hours	0	1	2	3	4	5	6
Rental Cost							

Make a graph of the data.

3. Explain how the cost per hour shows up in the table:

graph:

equation:

4. Explain how the 20 in the equation is represented in the table and in the graph.

5. Which of the following points satisfy the relationship represented by the equation? (0,4), (0,20), (7, 55). Explain your reasoning.

3.3 Summary

Focus Question: How do you calculate vales of y from an equation like $y= 3x + 5$ when values of x are given? How about $y = 5 + 3x$? When do you need such equations that involve two operations?

Investigation 3.4

The equation $p = 50 + 10n$ represents the relationship between the Wild World admission price p in dollars and the number of people n in a group. The right side of the equation $50 + 10n$ is an algebraic expression. It represents the value of the dependent variable, p. It involves two operations, addition and multiplication.

The critical question is "Which operation comes first?" Theo wants to find the admission price for an Ocean Bike Tours group with 17 members. He first works from left to right:

$$\begin{gathered} 50 + 10 \times 17 \\ = 60 \times 17 \\ = \mathbf{1{,}020} \end{gathered}$$

He gets a number that seems too large.

Then Theo enters the same expression on his calculator and gets:

$$50+10\cdot 17=\mathbf{220}$$

He puzzled by the difference in results. Then Theo remembers that there are rules for evaluating expressions.

- Which is the correct answer? Why?

Here are the rules known as the **Order of Operations**:

1. Work within parentheses
2. Write numbers written with exponents in standard form.
3. Do all multiplication and division in order from left to right.
4. Do all addition and subtraction in order from left to right.

Use the Order of Operations with $7 + (6 \times 4 - 9) \div 3$.

$$\begin{aligned} 7 + (6 \times 4 - 9) \div 3 &= 7 + (24 - 9) \div 3 \\ &= 7 + (15) \div 3 \\ &= 7 + 5 \\ &= 12 \end{aligned}$$

Practice the Order of Operations rules on these examples:

A. The group admission price at Wild World is given by the equation $p = 50+10n$. Find the prices for groups with 5, 11, and 23 members.

5:

11:

23:

B. The equation $b = 100 - 6r$ gives the number of pints left on a Wild World bonus card after r rides. Find the numbers of points left after:

3 rides:

7 rides:

14 rides:

C. Celia makes plans for the van ride home to Atlantic City from Williamsburg. She plans for a 2-hour stop in Baltimore, Maryland. To predict trip time t from average driving speed s, she writes the equation

$$t = 2 + \frac{350}{s}$$

Find the predicted trip times for average driving speeds of **45, 55, and 65** miles per hour.

45:

55:

65:

D. Sidney writes two equations: $I = 350n$ and $E = 150n + 1000$. The equations relate income I and operating expenses E to number of customers.

Sidney writes the equation $P = 350n - (150n + 1000)$ to show how tour profit P depends on the number of customers n. Use the rule to find profits P for:

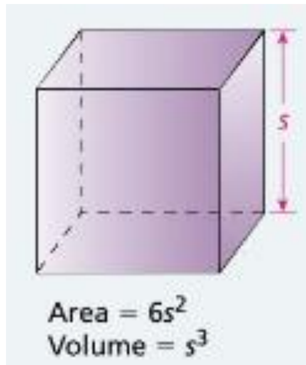
8 customers:

12 customers:

20 customers:

30 customers:

E. The Ocean Bike Tours partners have an Atlantic City workshop in the shape of a cube. The formula for the surface area of a cube is $A = 6s^2$. The formula for the volume of a cube is $V = s^3$.



1. If each edge of the cubical workshop is 4.25 meters long what is the total surface area of the floor, walls, and ceiling?

2. What is the volume of the workshop?

3.4 Summary

Focus Question: When an equation relating two variables involves two or more operations, how do you use the equation to find values of the dependent variable from given values of the independent variable?

Investigation 3 Reflection

In this Investigation, you wrote algebraic equations to express relationships between variables. You analyzed the relationships using tables and graphs. You also related the tables and graphs to the equations you wrote. The following questions will help you summarize what you have learned.

1. What strategies help in finding equations to express relationships?

2. For relationships given by equations in the form $y = mx$:

a. How does the value of y change as the value of x increases?

b. How is the pattern of change shown in a table, graph, and equation of the function?

Table:

Graph:

Equation:

3. In this Unit, you have represented relationships between variables with tables, graphs, and equations. List some advantages and disadvantages of each of these representations.

Tables:

Advantages:

Disadvantages:

Graphs:

Advantages:

Disadvantages:

Equations:

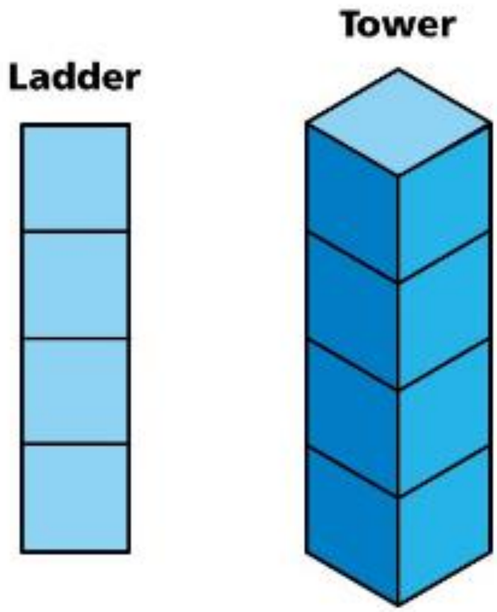
Advantages:

Disadvantages:

4. If the value of one variable in a relationship is known, describe how you can use a table, graph, or equation to find a value of the other variable.

Investigation 4.1

One of the most popular rides at Wild World is the Sky Dive. Riders are lifted in a car 240 feet in the air. When the car is released, it falls back to the ground. It reaches a speed near 50 miles per hour.



How many steel pieces do you need to build each of these figures?

Ladder:

Tower:

Suppose that you were building the tower for a similar ride.
How many steel pieces would you need to make a ladder of *n* squares?

How many steel pieces would you need to make a tower of *n* cubes?

A. 1. Look at the ladder of squares. Complete the table.

Number of Squares	1	2	3	4	5	10	20
Number of Pieces	4						

2. Write an equation that shows how to find the number of pieces *P* needed to make a ladder of *n* squares.

B. 1. Look at the tower of cubes. What numbers would go in the second row of a table that counts steel pieces needed to make a tower of *n* cubes?

Number of Cubes	1	2	3	4	5	10	20
Number of Pieces	12						

2. Write an equation that shows how to find the number of steel pieces in a tower of *n* cubes.

4.1 Summary

Focus Question: Is it possible to have two different, but equivalent, expressions for a given situation? Explain.

Investigation 4.2

A group of students worked on the ladder problem. Four of them came up with equations relating the number of steel pieces P to the number of squares n.

Tabitha: $P = n + n + n + 1$	Chaska: $P = 1 + 3n$
Latrell: $P = 4n$	Eva: $P = 4 + 3(n - 1)$

Recall that groups of mathematical symbols such as $n + n + n + 1$, $1 + 3n$, $4n$ and $4 + 3(n - 1)$ are called *algebraic expressions*. Each expression represents the value of the dependent variable P. When two expressions give the same results for every value of the variable, they are called **equivalent expressions**.

A. 1. What thinking might have led the students to their ideas?

Tabitha:

Latrell:

Chaska:

Eva:

2. Do the four equations predict the same numbers of steel pieces for ladders of any height n? Test your ideas by comparing values of P when $n = 1, 5, 0$ and 20.

3. Which of the expressions for the number of steel pieces in a ladder of *n* squares are equivalent? Why?

4. Are any of the expressions equivalent to your own from Problem 4.1? How can you be sure?

B. 1. Think about building a tower of cubes. Write two more expressions that are equivalent to the expression you wrote in part (2) of question B in Investigation 4.1. Explain why they are equivalent.

2. Pick two equivalent expressions from part (1). Use them to generate a table and graph for each. Compare the tables and graphs.

4.2 Summary

Focus Question: What does it mean to say that two algebraic expressions are equivalent?

Investigation 4.3

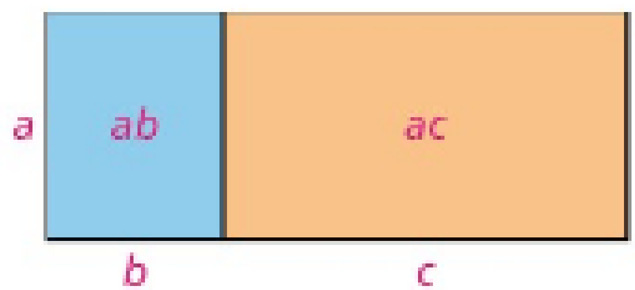
In an expression such as $1 + 3n$, the 1 and the $3n$ are called **terms** of the expression. In the expression $4 + 3(n – 1)$ there are 2 terms, 4 and $3(n - 1)$. Note that the expression $(n – 1)$ is both a factor of the term $3(n - 1)$ and a difference of two terms. The 3 is the **coefficient** of n in the expression $1 + 3n$.

The Distributive Property helps to show that two expressions are equivalent. It states that for any numbers a, b and c the following is true:

$$a(b + c) = ab + ac$$

This means that:

- A number can be expressed both as a product and as a sum.
- The area of a rectangle can be found in two different ways.

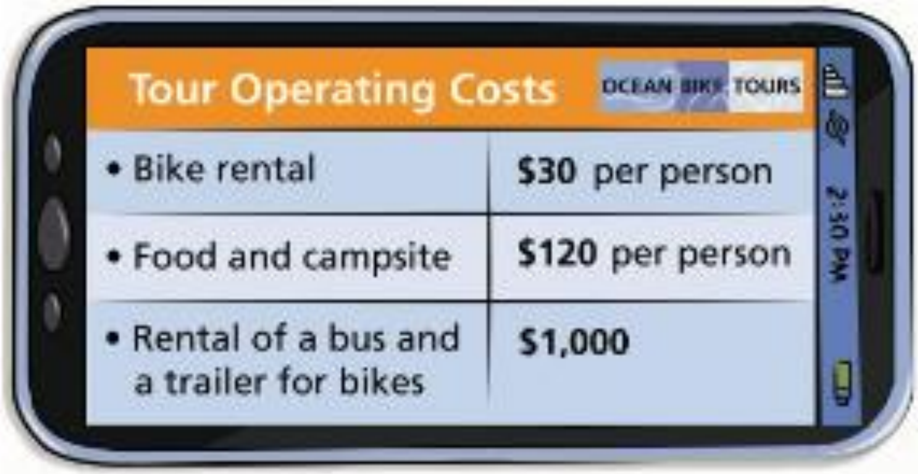


The expression $a(b + c)$ is in **factored form**.
The expression $a(b) + a(c)$ is in **expanded form**.
The expressions $a(b + c)$ and $ab + ac$ are **equivalent expressions**.

- Use the Distributive Property to write an equivalent expression for $5x + 6x$.

- How does this help write an equivalent expression for $n + n + n + 1$?

With their plans almost complete, the Ocean Bike Tours partners have made a list of tour operating costs.



Tour Operating Costs	OCEAN BIKE TOURS
• Bike rental	$30 per person
• Food and campsite	$120 per person
• Rental of a bus and a trailer for bikes	$1,000

What equation can represent the total costs?

Is there more than one possible equation? Explain.

The next step in planning is to write these costs as *algebraic expressions*.

A. What equations show how the three cost variables depend on the number of riders n?

1. Bike rental $B =$

2. Food and campsite fees $F =$

3. Rental of the bus and trailer $R =$

B. Three of the business partners write equations that relate total tour cost C to the number of riders n:

Celia's equation: $C = 30n + 120n + 1000$

Theo's equation: $C = 150n + 1000$

Liz's equation: $C = 1150n$

1a. Are any or all of these equations correct? If so, are they equivalent? Explain why.

b. For the equations that are correct, explain what information each term and coefficient represents in the equation.

2. Compare the equations. Use Order of Operations guidelines to complete the table below of sample (n, C) values. What does the table suggest about which expressions for C are equivalent?

Operating Cost Related to Number of Customers

Number of Customers n	**5**	**10**	**15**	**20**	**25**
$C = 30n + 120n + 1000$					
$C = 150n + 1000$ $C = 1150n$					

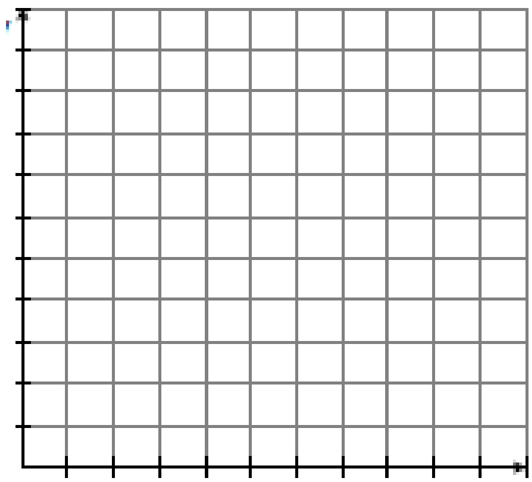
3. What results would you expect if you were to graph the three equations below?

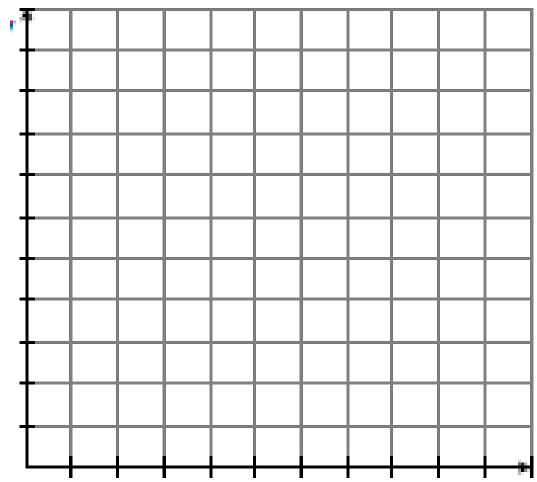
$$C = 30n + 120n + 1000$$

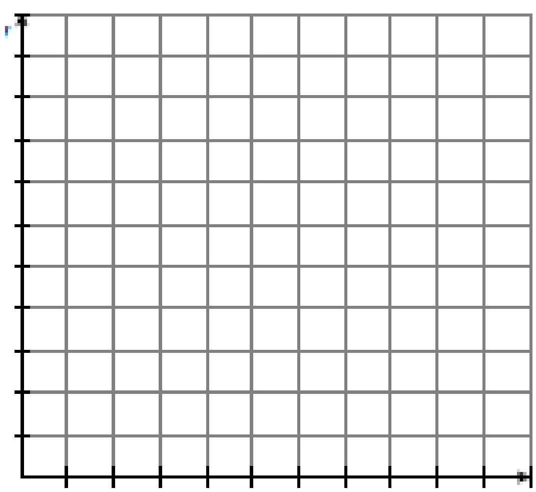
$$C = 150n + 1000$$

$$C = 1150n$$

Check your ideas by graphing.







4. Use properties of operations such as the Distributive Property to show which expressions for cost are equivalent.

C. 1. For each expression below, circle the terms and box the coefficient in each term.

a. $5x + x + 6$ **b**. $10q - 2q$

2. Use the properties of operations to write an equivalent expression for each expression above.

3. Show that $1 + 3n = 4 + 3(n - 1)$

D. Sidney points out that all three partners left out the cost of the Wild World Amusement Park trip. The cost for that part of the tour is $W = 50 + 10n$. How does this cost factor change each correct equation? (Rewrite the equations next to the original equations in B.3.)

4.3 Summary

Focus Question: How can expressions such as $3x + 7x$ or $3(x + 2)$ be written in equivalent form?

Investigation 4.4

The Ocean Bike Tours partners decide to charge $350 per rider. This leads them to an equation giving tour income I for n riders: $I = 350n$. You can use the equation to find the income for 10 riders.

$$I = 350n$$
$$I = 350 \times 10$$
$$I = 3{,}500$$

Suppose you are asked to find the number of riders needed to reach a tour income goal of $4,200. In earlier work you used tables and graphs to estimate answers. You can also use the equation: $4{,}200 = 350n$

Solving the equation means finding values of n that makes the equation $4{,}200 = 350n$ a true statement. Any values of n that work are called **solutions of the equation**.

One way to solve equations is to think about the fact families that relate arithmetic operations. Examples:

$5 + 7 = 12$
$5 = 12 - 7$

Both equations describe true relationships between 5, 7, and 12.

$5(7) = 35$
$5 = 35 \div 7$

Both equations describe true relationships between 5, 7, and 35.

- How are fact families helpful to solve equations such as $c = 350n$?

When you find the solution of an equation, it is always a good idea to check your work.

Is n = 12 a solution for 4200 = 35n?

Substitute 12 for n; 4,200 = 35(12).

Is this a true statement?

Multiplying 35 by 12 equals 4,200.

Yes, 12 is the solution.

A. Single admissions at Wild World Amusement Park cost \$21. If the park sells n single admissions in one day, its income is $I = 21n$.

1. **Write an equation to answer this question**:
How many single admissions were sold on a day the park had income of \$9,450 from single admissions?

2. Solve the equation. Explain how you found your answer.

3. How can you check your answer?

B. On the Ocean Bike Tours test run, Sidney stopped the van at a gas station. The station advertised 24 cents off per gallon on Tuesdays.

1. Write an equation for the Tuesday discount price d. Use p as the price on other days.

2. Use the equation to find the price on days other than Tuesday if the discount price is \$2.79.

C. Ocean Bike Tours wants to provide bandanas for each person. The cost of the bandanas is $95.50 for the design plus $1 per bandana.

1. Write an equation that represents this relationship.

2. Use the equation to find the cost for 50 bandanas.

3. Use the equation to find the number of bandanas if the total cost is $116.50.

In Questions A-C you wrote and solved equations that match questions about the bike tour. Knowing about the problem situation often helps in writing and solving equations. But the methods you use in those cases can be applied to other equations without stories to help your reasoning.

D. Use ideas you've learned about solving equations to solve the equations below. Show your calculations. Check each solution in the equation.

1. $x + 22.5 = 49.25$

Check:

2. $37.2 = n - 12$

Check:

3. $55t = 176$

Check:

4.4 Summary

Focus Question: What strategies can you use to solve equations in the forms $x + a = b$, $x - a = b$, $ax = b$, and $x \div a = b$ $(a \neq 0)$?

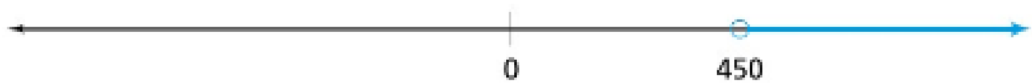
Investigation 4.5

In each part of Investigation 4.4 you wrote and solved an equation about ocean Bike Tours. For example, you wrote the equation $21I = C$. Then you were told that income was \$9,450. You solved the equation $24I – 9,450$ to find the number of riders. The solution was $I = 450$.

Suppose you were asked a related question: How many single-admission sales will bring income of more than \$9,450?

To answer this question, you need to solve the inequality $21I > 9,450$. That is, you need to find values of the variable I that make the given inequality true. This task is very similar to what you did when comparing rental plans offered by the two bike shops in Investigation 2.1.

If $21I = 9,450$, then $I = 450$. So, any number $I > 450$ is a solution in the inequality $21I > 9,450$. A graph of these solutions on a number line is:

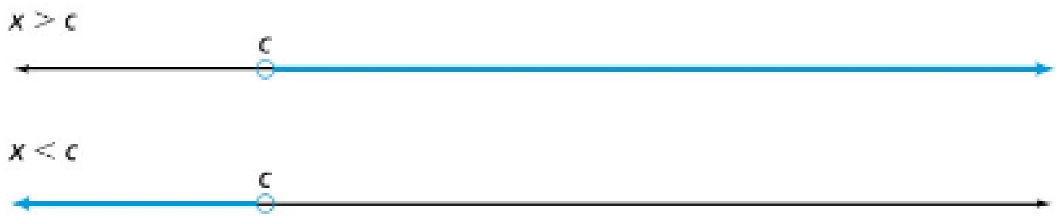


- What are five possible solutions for I?

- What are five more possible solutions for I?

- How many possible solutions does this inequality have?

- In general, the solution to a simple inequality can be written in the form $x > c$ or $x < c$. Those solutions can be graphed on a number line. Below are two examples.



What does the thicker part of each number line tell you about solutions to the inequality?

Use what you know about variables, expressions, and equations to write and solve inequalities that match Questions A-C. In each case, do the following>

- Write an inequality that helps to answer the question.
- Give at least 3 specific number solutions to the inequality. Then explain why they work.
- Describe all possible solutions.

A. The bungee jump at Wild World charges \$35. How many jumpers are needed for the jump to earn income of more than \$1,050 in a day?

B. A gas station sign says regular unleaded gasoline costs \$4 per gallon. How much gas can Mike buy if he has \$17.50 in his pocket?

C. Ocean Bike Tours wants to provide bandanas for each customer. The costs are \$95.50 for the design plus \$1 per bandana. How many bandanas can they buy if they want the cost to be less than \$400?

D. Use ideas about solving equations and inequalities from Questions A, B, and C to solve the inequalities below.

1. $84 < 14m$

2. $55t > 176$

3. $x + 22.5 < 49.25$

4. $37.2 > n - 12$

E. 1. Make up a problem that can be represented by the equation $y = 50 + x$.

2. Which of these points lie on the graph of the equation? (8, 92), (15, 110)

3. Use a point that lies on the graph to make up a question that the point can answer.

4. Use a point that lies on the graph to write an inequality that the point satisfies.

4.5 Summary

Focus Question: How can you represent and find solutions for inequalities?