Comparing Bits and Pieces
Ratios, Rational Numbers, and Equivalence

Name: __________________  Hour: ___
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1.1 Investigation

Students at a middle school are organizing three fundraising projects to raise money. The eighth grade will sell calendars. The seventh grade will sell popcorn. The sixth grade will sell posters.

Each grade picks a different goal for its fundraiser. The three grades are competing to see which grade will reach its fundraising goal first.

The fundraising goal for each grade is displayed on a banner in front of the principal’s office.

Today you will look at different claims (a statement based on information believed to be true) made by students about the fundraising goals above. You will need to determine whether you think each claim is true or false and then explain how you know this. Begin looking for similarities in the types of comparisons present.
A. The students wrote some claims about the fundraising goals on slips of paper and gave them to the principal to read over the loudspeaker during the morning announcements. Decide whether each claim is true. Explain your reasoning.

Markus:
The sixth-grade goal is $150 more than the eighth-grade goal.

Kimi:
When the sixth graders meet their goal, they will have raised $\frac{2}{3}$ of the seventh-grade goal.

Lakisha:
The eighth-grade goal is half the sixth-grade goal.

Andres:
For every dollar the eighth graders plan to raise, the sixth graders plan to raise two dollars.
Ben:
For every $60 the sixth graders plan to raise, the seventh graders plan to raise $90.

Eliza:
The sixth-grade goal is 200% of the eighth-grade goal.

Chung:
For every $3 the eighth grade plans to raise, the seventh grade plans to raise $1.

B. Write three more true comparison statements for the principal to read over the loudspeaker.

1.

2.

3.
C. On the first day of the fundraiser, the principal announces one more goal over the loudspeaker – the teachers’ fundraising goal. The microphone is not working very well. What do you think the teachers’ goal is?

Good morning students, teachers, and staff! The teachers have joined the school fundraiser. They will be selling books for summer reading. They have set a goal of \( \text{STATIC} \) dollars. This is 210 dollars more than the \( \text{STATIC} \) graders, but only \( \frac{4}{5} \) as much as the \( \text{STATIC} \) graders. For every 60 dollars the teachers plan to raise, the \( \text{STATIC} \) graders plan to raise 50 dollars.
1.1 Summary

___________________ Comparisons
What does it mean?

Examples:

___________________ Comparisons
What does it mean?

Examples:
___________________
___________________
___________________

Focus Question: What are two ways to compare a $500 fundraising goal to a $200 fundraising goal?
1.2 Investigation

The ratio of the sixth-grade goal to the seventh-grade goal is _____ to _____.

or

_____ to _____

or

_____ to _____

or

_____ to _____

These are called ______________________   _____________ because…

Is it true that the ratio of the sixth-grade goal to the seventh-grade goal is 90 to 60?
The principal labeled some of the marks on the four thermometers with dollar amounts. Decide what labels belong on the remaining marks.
A. 1. Ben said: *For every $60 the sixth graders plan to raise, the seventh graders plan to raise $90.* He looks at the principal’s thermometers and sees that $60 is at the same place on the sixth-grade thermometer as $90 is on the seventh-grade thermometer.

Ben also makes the claim: *For every $30 the sixth graders plan to raise, the seventh graders plan to raise $45.*

Do you agree with this claim? Explain your reasoning.

2. Use the thermometers to write two more *for every* claims that relate the fundraising goals.
   1. For every_______________________________________________________
      __________________________________________________________
      ________________________________________________________ .

   2. For every _______________________________________________
      _________________________________________________________
      ________________________________________________________ .
B. You can write each of the comparisons in Question A as a ratio. A ratio is a kind of comparison.

Here are two ways that you can rewrite the comparisons in Question A.

The ratio of the sixth-grade goal to the seventh-grade goal is 60 to 90.

The ratio of the sixth-grade goal to the seventh-grade goal is 30 to 45.

Rewrite your comparisons from Question A using the word ratio instead of "for every" Be ready to share out!

1. The ratio of ____________________________________________

________________________________________________________

________________________________________________________.

2. The ratio of ____________________________________________

________________________________________________________

________________________________________________________.

C. 1. Equivalent ratios are ratios with different numbers that are equal or show the same relationship. List some other pairs of equivalent ratios you have found in this Investigation.

2. What patterns do you notice in your ratios that can help you find other equivalent ratios?
Focus Question: How does a “for every” statement show a ratio comparison?
1.3 Investigation

C. 1. Some students began to make a number line using their one-third, one-sixth, one-ninth, and one-twelfth fraction strips. The drawing shows their work so far. One student used the top fraction strip to mark \( \frac{2}{3} \) on the number line.

Name 3 fractions equivalent to 2/3 from the picture above.

1. 

2. 

3. 

Now using your number line find 3 fractions equivalent to 1/3

1. 

2. 

3. 

What relationship do you see between the equivalent fractions in each of the previous questions?
D. Some other students began to mark a number line using different fraction strips. Use their drawings to measure distances between points. For example, the distance between the mark labeled 0 and the mark labeled $\frac{3}{5}$ is $\frac{3}{5}$.

1. What is the distance between each pair of points?
   a. 0 and $\frac{7}{10}$
   c. $\frac{7}{10}$ and 1
   b. $\frac{3}{5}$ and $\frac{7}{10}$
   d. $\frac{3}{5}$ and 1

2. What is the distance between each pair of points?
   a. 0 and $\frac{1}{3}$
   d. $\frac{1}{2}$ and $\frac{2}{3}$
   b. $\frac{1}{3}$ and $\frac{1}{2}$
   e. $\frac{1}{2}$ and 1
   c. $\frac{1}{3}$ and $\frac{2}{3}$
   f. $\frac{2}{3}$ and 1
E. 1. How can fraction strips, number lines, and thinking with numbers help you find equivalent fractions?

2. Matt claims that $\frac{1}{3}$ can indicate a point on a number line as well as distance. Is he correct? Explain.

3. Sally said that the fraction strips remind her of rulers and that you could use fraction strips to measure the progress on the fundraising thermometers. What do you think?
1.3 Summary

**Focus Question:** When you fold fraction strips, what relationships do you see emerge that show how the numerator and denominator change to make equivalent fractions?

How can we find equivalent fractions?

1. _______________ ____________ and _____________ ___________  
2.  
3.  
4.  

15
1.4 Investigation

Fractional Parts:

\[
\frac{3}{5}
\]

How can you use factors and multiples to write equivalent fractions?
Ben and Kimi are each comparing one sixth-grade goal to one seventh-grade goal. Ben uses ratios to make comparisons and Kimi uses fractions to make comparisons. Think about some ways in which working with fractions is like and not like working with ratios.

When you use fractions to compare a part to a whole, you often have more than one fraction name for the same quantity. For example, in Investigation 1.3, you found that \( \frac{1}{5} = \frac{2}{10} \).

In this next Investigation, you will compare the fundraising progress of a grade to its fundraising goal using fractions.

The thermometers on the next page show the progress of the sixth-grade poster sales after 2, 4, 6, 8 and 10 days. The principal needs to know what fraction of the goal the sixth grade has achieved after each day.
A. How can you tell whether the sixth graders raised the same amount each day? Explain.

B. Decide what fraction of their goal the sixth graders had raised after

Day 2? Day 8?
Day 4? Day 10?
Day 6?

C. 1. Mary used her fourths strip to measure and label fractions and dollar amounts on the Day 2 thermometer at the right. Did she write the correct dollar amounts? How do you know?

2. Use your fraction strips to measure and label fraction and dollar amounts on the copies of the remaining thermometers.
D. 1. Jeri says that she can express the sixth-graders’ progress on Day 2 in two ways using equivalent fractions: \( \frac{1}{4} = \frac{2}{8} \) of the goal. Find some other days for which you can write the sixth-graders’ progress with two or more equivalent fractions.

2. Why do \( \frac{1}{4} \) and \( \frac{2}{8} \) both correctly describe the sixth-graders’ progress on Day 2?

E. At the end of Day 9, the sixth graders have raised $240.
   1. What fraction of their goal have they reached?

2. Using the blank thermometer to the right, show how you would shade the progress made for Day 9.

In C. 2., you were asked to use your fractional amounts to determine the total amount of money they raised each day. How did you do this? Explain your strategy. LOTS OF DETAIL!!!! We are going to share these as a class.
Focus Question: How can fraction strips help you to find part of a number?
1.5 Investigation

What is a ratio?

Ways to write a ratio:

1.

2.

3.
What do you notice in these thermometers?
A. 1. What fraction of its goal did each grade reach by the end of Day 10 of the fundraiser?

6th:

7th:

8th:

Teachers:

2. How much money did each group raise? (SHOW YOUR WORK)

6th:

7th:

8th:

Teachers:
B. Margarita said: “I think the seventh graders raised $300 by the end of Day 10 because I wrote several fractions that are equivalent to what I found with my fraction strips: $\frac{2}{3}$.”

\[
\frac{2}{3} = \frac{4}{6} = \frac{20}{30} = \frac{60}{90} = \frac{300}{450}
\]

Margarita also drew this picture.

1. Explain how Margarita found these equivalent fractions. How does her picture relate to her method of finding equivalent fractions?

2. Use equivalent fractions to show how much money the sixth graders had raised by the end of Day 10.

3. Use equivalent fractions to show how much money the teachers had raised by the end of Day 10.
C. 1. Brian wrote this comparison statement: The ratio of the amount of money raised by the sixth graders to the amount raised by the seventh graders is 250 : 300. Is this a correct statement? Explain.

2. Kate thought of $250 as 25 ten-dollar bills and $300 as 30 ten-dollar bills. She wrote the ratio, 25 : 30. Write a comparison statement using Kate’s ratio.

3. Are Brian and Kate’s two ratios equivalent? Explain.

4. What ratio would Kate write if she thought of $250 and $300 as numbers of fifty-dollar bills? Would thinking of twenty-dollar bills work? Explain.

5. Write two comparison statements, using equivalent ratios, for amounts of money raised by the sixth grade compared to the eighth grade in the fundraiser.
1.5 Summary

**Focus Question:** What does it mean for two fractions to be equivalent?

**Focus Question:** What does it mean for two ratios to be equivalent?

On the last day of the fundraiser, the principal announces the results using both fractions and ratios. She has these two sticky notes on her desk.

![Sticky notes showing equivalent fractions and ratios](image)

1. What do you think is the meaning of each note?

2. When are fractions useful?

3. When are ratios useful?
2.1 Investigation

Ratio statements can also be written as “per” statements. For example, “It costs $120 per 10 students to go on the trip.” An equivalent comparison statement is “the cost per student to go on a field trip is $12.” Now you can say

\[
\begin{align*}
$12 & \text{ for every 1 student} \\
$12 & \text{ for each student} \\
$12 & \text{ per student}
\end{align*}
\]

This particular comparison, cost per one student, is called a unit rate. A unit rate is a comparison in which one of the numbers being compared is 1 unit.

If Mrs. Nussdorfer earns 24 dollars for every 4 hours she works, how much does she make PER hour? Show your work.

Introducing Unit Rate

Often we share food so that each person gets the same amount. This may mean that food is cut into smaller pieces. Think about how to share a chewy fruit worm that is already marked in equal-sized pieces.

The chewy fruit worm below shows four equal segments.
A. 1. Show two ways that four people can share a 6-segment chewy fruit worm. In each case, how many segments does each person get?

2. Show two ways that six people can share an 8-segment chewy fruit worm. In each case, how many segments does each person get?
B. 1. Show how 12 people can share an 8-segment chewy fruit worm. How many segments are there for every person?

2. Show how five people can share a 3-segment chewy fruit worm. How much is this per person?

C. Jena wants to share a 6-segment chewy fruit worm. The tape diagram below shows the marks she made on the worm so she can share it equally among the members in her group.

1. How many people are in her group?

2. Is there more than one possible answer to part 1? Explain.

3. What is the number of segments per person?

4. Write a fraction to show the part of the chewy fruit worm each person gets.
D. Would you rather be one of four people sharing a 6-segment chewy fruit worm or one of eight people sharing a 12-segment chewy fruit worm? Explain.

2.1 Summary

Focus Question: What does a unit rate comparison statement tell us? Describe how you found or used unit rates.
2.2 Investigation

Sometimes there are reasons to share quantities *unequally*. Suppose your older brother paid more than half the cost of a video game. You might think it is fair for him to spend more time playing the game. At a party, you might agree that your friend should take the bigger piece of chocolate cake because your friend likes chocolate more than you do.

Two sisters, Crystal and Alexa, are going to a strange birthday party. Instead of birthday cake, pairs of party guests are each served a large chewy fruit worm to share according to their ages. Since the sisters are not the same age, they do not share their fruit worm equally.

Crystal is 12 years old and Alexa is 6 years old. Their chewy fruit worm has 18 segments. According to their ages, Crystal gets 12 segments and Alexa gets 6 segments. The ratio of the girls’ shares of the worm, 12 to 6, is equivalent to the ratio of their ages, 12 to 6.

```
C C C C C C C C C C A A A A A A A
```

- According to the rule, how would the girls share a 9-segment chewy fruit worm?

Since Crystal's age is two times Alexa's age, Crystal gets twice as many segments as Alexa. The ratio of Crystal's segments to Alexa's segments is 12 to 6 or 2 to 1.

- The ratio 2 to 1 is a unit rate. What do the numbers 2 and 1 mean for the sisters?
A. Draw some chewy fruit worms with different numbers of segments that Crystal and Alexa can share without having to make new cuts.

B. 1. Jared is 10 years old. His brother Peter is 15 years old. What are some chewy fruit worms they can share without having to cut up segments? Draw an example using a tape diagram and then write the ratio comparing the number of segments Jared gets to the number of segments Peter gets.

Worm 1 Ratio:

Worm 2 Ratio:

Worm 3 Ratio:
2. Are the ratios you write in part 1 equivalent to each other? Explain.

3. How would you write a unit rate to compare how many segments Jared and Peter get?

C. 1. Caleb and Isaiah are brothers. They share a 14-segment chewy fruit worm according to their age. How old could they be?

2. Caleb gets 8 out of the 14 segments of the chewy fruit worm, so he gets \( \frac{8}{14} \) and Isaiah gets \( \frac{6}{14} \) of the worm.

   a. From Question A, what fractions of the chewy fruit worm do Crystal and Alexa each get at the birthday party?

      Crystal: Alexa:

   b. From Question B, what fractions of the chewy fruit worm do Jared and Peter each get at the birthday party?

      Jared: Peter

   c. How does the ratio of segments that Caleb and Isaiah get relate to the fractions of the chewy fruit worm that they each get?
2.2 Summary

**Focus Question:** Why is the ratio of Crystal’s part to Alexa’s part 12:6 but Crystal’s part to both girls is 12:18?

Caleb and Isaiah example:
2.3 Investigation

When comparing how to share chewy fruit worms, Crystal recorded how many segments she and her sister would get for different sizes of chewy fruit worms. Crystal thought she could use what she knew about equivalence to make a table showing the amounts.

### Comparing Segments

<table>
<thead>
<tr>
<th>Segments for Alexa</th>
<th>6</th>
<th>3</th>
<th>1</th>
<th>2</th>
<th>$\frac{1}{2}$</th>
<th>10</th>
</tr>
</thead>
<tbody>
<tr>
<td>Segments for Crystal</td>
<td>12</td>
<td>6</td>
<td>2</td>
<td>4</td>
<td>1</td>
<td>20</td>
</tr>
</tbody>
</table>

The table shows that for every segment given to Alexa, Crystal gets two segments. This is Alexa's unit rate. The table also shows that for every $\frac{1}{2}$ segment Alexa is given, Crystal gets one segment. This is Crystal's unit rate.

Crystal sees an ad for chewy fruit worms. She decides she wants the student council to include chewy fruit worms in the fundraising sale.

You can use the information in the advertisement to compute the price for any number of worms you want to buy. One way to figure out the price of a single item from a quantity price is use the information to build a **rate table** of equivalent ratios.

The rate table in Question A shows the price for different numbers of chewy fruit worms. The cost of 30 chewy fruit worms is $3.
A. 1. Crystal wants to calculate costs quickly for many different numbers of chewy fruit worms. Complete the rate table below with prices for each of the numbers of chewy fruit worms.

<table>
<thead>
<tr>
<th>Number of Worms</th>
<th>1</th>
<th>5</th>
<th>10</th>
<th>15</th>
<th>30</th>
<th>90</th>
<th>150</th>
<th>180</th>
</tr>
</thead>
<tbody>
<tr>
<td>Reduced Price</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>$3</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

2. How much do 3 chewy fruit worms cost? 300 chewy fruit worms?

3. How many chewy fruit worms can you buy for $50? For $10?

4. What is the unit price of one chewy fruit worm? What is the unit rate?

B. The student council also decides to sell popcorn to raise money. One ounce of popcorn (unpopped) kernels yields 4 cups of popcorn. One serving is a bag of popcorn that holds 2 cups of popcorn.

1. Use a rate table to find the number of ounces of popcorn kernels needed to determine the cups of popcorn.

<table>
<thead>
<tr>
<th>Number of Cups of Popcorn</th>
<th>4</th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>Number of Ounces of Popcorn Kernels</td>
<td>1</td>
<td>2</td>
<td>3</td>
<td>4</td>
<td>5</td>
<td>6</td>
<td>7</td>
<td>8</td>
</tr>
<tr>
<td></td>
<td>9</td>
<td>10</td>
<td>11</td>
<td>12</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
2. How many cups of popcorn can you make from 12 ounces of popcorn kernels? From 30 ounces of popcorn kernels?

3. How many ounces of popcorn kernels are needed to make 40 cups of popcorn? To make 100 cups of popcorn?

4. How many ounces of kernels are needed to make 100 servings?

5. How many ounces of kernels are needed to make 1 cup?

C. 1. How do rate tables help you answer Question A and Question B?

2. How do unit rates help you answer Question A and Question B?
2.3 Summary

**Focus Question:** How do rate tables help us find equivalent ratios?

Why are unit rates helpful?
3.1 Day 1 Investigation
Extending the Number Line: Integers and Mixed Numbers

In Investigation 1, you worked with the part of the number line between 0 and 1, shown below.

![Number Line Diagram](image)

The whole numbers on a number line follow one another in a simple, regular pattern. Between every pair of whole numbers are many other points that may be labeled with fractions.

A number such as $1\frac{1}{2}$ is called a **mixed number** because it has a whole number part and a fraction part. Another way to write this number is as an **improper fraction**. For positive numbers, an **improper fraction** such as $\frac{3}{2}$ has a numerator greater than or equal to the denominator.

![Number Line Diagram](image)

- Why can this point be labeled with two names: $1\frac{1}{2}$ and $\frac{3}{2}$?

There is really nothing improper about these fractions. This is just a name used for fractions that represent more than one whole. You may have used a mixed number or an improper fraction to express the teachers’ fundraising success in the previous Investigation.
The number line can be extended in both directions, as shown below. Numbers to the left of zero are marked with a “−” sign and are read as negative one, negative two, etc.

In this Problem, you will use fractions, mixed numbers, and improper fractions. You can represent positive and negative fractions and mixed numbers as points on the number line.

- Betty says that the mark between 2 and 3 should be labeled $\frac{1}{2}$. Do you agree?

- Judi says that the mark between 2 and 3 should be labeled $\frac{5}{2}$. Do you agree?

- What label should you put on the mark between −2 and −3?

- If there were a mark halfway between that mark and −2, what label would you put on it?
On the number line below, 5 and \(-5\) are the same distance from 0 but in opposite directions. Therefore, 5 and \(-5\) are opposites. The opposite of 5 is \(-5\). The opposite of \(-5\) is 5. Similarly, the opposite of \(2\frac{1}{2}\) is \(-2\frac{1}{2}\), and the opposite of \(-2\frac{1}{2}\) is \(2\frac{1}{2}\).

**A 1.** On a number line like the one below, mark and label these fractions.

\[
\begin{array}{cccccccccccc}
\frac{1}{4} & \frac{2}{4} & \frac{3}{4} & \frac{4}{4} & \frac{5}{4} & \frac{6}{4} & \frac{7}{4} & \frac{8}{4} & \frac{9}{4} & 0 & -\frac{1}{4} & -\frac{2}{4} & -\frac{3}{4} & -\frac{4}{4} & -\frac{5}{4}
\end{array}
\]

2. Which of the fractions can be written as mixed numbers? Explain.
3. What is the opposite of $\frac{1}{2}$?

4. What is the opposite of $-\frac{1}{2}$?

5. What is the opposite of zero?

6. How far away is $-9$ from zero on the number line?

7. How far away is 9 from zero on the number line?

8. How are the distances of 9 and $-9$ from zero alike?
3.1 Day 2 Investigation

The **absolute value** of a number is its distance from 0 on the number line. Numbers that are the same distance from 0 have the same absolute value. The absolute value of \(2\frac{1}{2}\) and the absolute value of \(-2\frac{1}{2}\) are both \(2\frac{1}{2}\).

You can express the absolute value of a number two ways without words.

\[
\begin{align*}
|2\frac{1}{2}| &= 2\frac{1}{2} \\
|-2\frac{1}{2}| &= 2\frac{1}{2}
\end{align*}
\]

OR

\[
\begin{align*}
\text{abs}(2\frac{1}{2}) &= 2\frac{1}{2} \\
\text{abs}(-2\frac{1}{2}) &= 2\frac{1}{2}
\end{align*}
\]

- What is the opposite of \(-\frac{2}{3}\)? What is the opposite of \(\frac{2}{3}\)?
- What is the absolute value of \(-\frac{2}{3}\)? What is the absolute value of \(\frac{2}{3}\)?

Zero, whole numbers, fractions, and their opposites are **rational numbers**. The numbers \(-\frac{9}{5}\), \(-3\), 0, \(\frac{2}{3}\), and \(2\frac{1}{3}\) are all rational numbers.

![Rational Numbers Diagram](image)

Negative numbers can also be improper fractions. Improper fractions have an absolute value greater than or equal to 1. Both \(\frac{7}{5}\) and \(-\frac{7}{5}\) are improper fractions. They can be written as \(1\frac{2}{5}\) and \(-1\frac{2}{5}\).

**D. 1.** What numbers have an absolute value of 1?

**2.** How many numbers have an absolute value of \(\frac{5}{4}\)?

What are the numbers?
3. How many numbers have an absolute value of 0?

1. a. Griffin visited her grandfather in Canada twice in the same year. During those visits, her grandmother took pictures of Griffin with her grandfather. Griffin says the absolute value of the temperature each day was 10. Is this possible? Explain. What is the difference between the two temperatures in degrees?

b. Griffin says the bird’s height above and the fish’s depth below sea level are opposites. Is this possible? Explain.
2. Aaron is playing a game in which he earns points for a correct answer and loses the same number of points for an incorrect answer.

   a. Aaron has zero points. The next question is worth 300 points. Aaron says, “It doesn’t matter whether I get the answer right or wrong, the absolute value of my score will be 300.” Do you agree? Why or why not?

b. Later in the game, Aaron’s score is back to zero. He then answers two more questions and his score is back to zero again. What could be the point values of the last two questions?

3.1 Summary

**Focus Question:** How can the number line help you think about fractions greater than 1 and less than 0?
3.2 Investigation

Estimating and Ordering Rational Numbers

When you solve problems involving fractions and decimals, you may find it useful to estimate the size of the numbers. One way is to compare each positive fraction to 0, \( \frac{1}{2} \), and 1. These numbers serve as benchmarks, or reference points. You also can compare each negative fraction to the opposites of the benchmarks: 0, \(-\frac{1}{2}\), and \(-1\). These benchmarks divide the number line below into six equal intervals: the interval between \(-1\frac{1}{2}\) and \(-1\), the interval between \(-1\) and \(-\frac{1}{2}\), and so on.

How can you decide which benchmark is closest to a given rational number?
A. Using your fraction cards, determine which interval each fraction falls between and place the card in the correct interval that represents what each fraction is closest to. Record your work on the diagram below.
B. Insert a less than (<), greater than (>), or equal to (=) symbol in each sentence below. Explain how the numbers or the number line helped you decide.

1. \(-\frac{5}{2}\) \(\pm\) 3
2. 0 \(\pm\) -3
3. \(-\frac{5}{3}\) \(\pm\) \(-\frac{11}{2}\)

4. Callum says that every number is greater than its opposite. Do you agree? Explain.

5. Blake says that he can use the absolute value to help order the numbers \(-\frac{6}{5}\) and \(-\frac{2}{3}\). He says the absolute value of \(-\frac{6}{5}\) is greater so it is farther away from zero, and therefore \(-\frac{6}{5}\) < \(-\frac{2}{3}\). Do you agree? Explain.

6. Will Blake’s strategy work for all of the comparisons you did in Questions 1-3? Explain.

C. Compare each pair of fractions using benchmarks and other strategies. Then copy the fractions and insert a less than (<), greater than (>), or equal to (=) symbol. Describe your strategies.

1. \(\frac{5}{8}\) \(\pm\) \(\frac{6}{8}\)
2. \(\frac{5}{6}\) \(\pm\) \(\frac{5}{8}\)
3. \(\frac{2}{3}\) \(\pm\) \(\frac{3}{9}\)
4. \(\frac{13}{12}\) \(\pm\) \(\frac{6}{5}\)
5. \(\frac{3}{4}\) \(\pm\) \(\frac{2}{5}\)
6. \(-\frac{1}{5}\) \(\pm\) \(-\frac{1}{3}\)
D. The smartphone screen shows deposits to and withdrawals from Brian’s checking account.

1. Which account activities have the same absolute value? What information does this provide for Brian?

2. Brian says that he spent less money on October 27th than he did on October 21st because the absolute value of the account withdrawal is closer to zero. Do you agree? Explain.

3.2 Summary

Focus Question: When comparing two rational numbers, what are some useful strategies for deciding which is greater?
3.3 Day 1 Investigation

Sharing 100 Things: Using Tenths and Hundredths

We see decimals in all different places of the world. What are the different places you see decimals in the world around you?

Decimals give people a way to write fractions with denominators of 10 or 100 or 1,000 or 10,000 or even 100,000,000,000, as in the table below. These denominators are different forms of base ten numeration.

<table>
<thead>
<tr>
<th>Fraction</th>
<th>Denominator as a Power of 10</th>
<th>Decimal</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\frac{1}{10}$</td>
<td>$\frac{1}{10^1}$</td>
<td>0.1</td>
</tr>
<tr>
<td>$\frac{1}{100}$</td>
<td>$\frac{1}{10^2}$</td>
<td>0.01</td>
</tr>
<tr>
<td>$\frac{1}{1,000}$</td>
<td>$\frac{1}{10^3}$</td>
<td>0.001</td>
</tr>
<tr>
<td>$\frac{1}{10,000}$</td>
<td>$\frac{1}{10^4}$</td>
<td>0.0001</td>
</tr>
<tr>
<td>$\frac{1}{100,000}$</td>
<td>$\frac{1}{10^5}$</td>
<td>0.00001</td>
</tr>
<tr>
<td>$\frac{1}{1,000,000}$</td>
<td>$\frac{1}{10^6}$</td>
<td>0.000001</td>
</tr>
<tr>
<td>$\frac{1}{100,000,000,000}$</td>
<td>?</td>
<td>?</td>
</tr>
</tbody>
</table>
How can you shade a grid to show the following?

<table>
<thead>
<tr>
<th>Fraction</th>
<th>Decimal</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \frac{20}{100} )</td>
<td>0.2</td>
</tr>
<tr>
<td>( \frac{250}{1,000} )</td>
<td>0.20</td>
</tr>
</tbody>
</table>

- Do 0.20, 0.02, and 0.2 all represent the same number? Explain.

- What are some fractions and decimals equivalent to \( \frac{3}{10} \)?
Wendy’s mother Ann makes lasagna every year to celebrate the winter holiday season. She makes the lasagna in an enormous 20 inch-by-20 inch square pan.

Ann cuts the lasagna into 100 servings to share with friends, family, neighbors, and co-workers. You may use grids to help answer the following questions.

Ann gave one pan of lasagna to her ten co-workers.

1. If the co-workers share the lasagna equally, how many servings will each co-worker get? Write each person’s share as a fractional and a decimal part of a pan.

Servings:

Fractional Part:

Decimal Part:

2. Ann’s pan is 20 inches by 20 inches. Describe each co-worker’s share of the lasagna.
B. Ann baked three more pans of lasagna. Each pan was shared with a different group of people. For each group below,

- write each person’s share as a number of servings.
- write each person’s share as a fractional and decimal part of a pan.

1. One pan was shared among Wendy’s four favorite teachers.

2. Ann miscalculated and had only one pan to share among all 200 sixth-graders at Wendy’s school.

3. One pan went to eight of Ann’s neighbors.
3.3 Day 2 Investigation

Sonam compared the decimal numbers 0.1 and 0.09 by thinking about lasagna. She said that 0.1 represents one serving of Ann’s lasagna and 0.09 represents 9 servings, so $0.09 > 0.1$. What do you think? Explain your reasoning.

Example Investigation 1

Write a fraction equivalent to 0.64 then explain how you figured this out.

Is there a rule you could use to find any decimal as a fraction?
Example Investigation 2

Write a decimal equivalent to the fraction \( \frac{3}{5} \) then explain how you figured this out.

Is there a rule you could use to write any fraction as a decimal?

Example Investigation 3

Which is bigger 0.3 or 0.30? Explain how you know!

3.3 Summary

Focus Question: How does what you know about fractions help you understand decimals?
3.4 Day 1 Investigation

Decimals on the Number Line

In Problem 3.3, you thought about a pan of lasagna that was cut into 100 pieces, and you considered relationships between the fractional and decimal representations for different numbers of servings.

The place value chart on the next page shows the names of each position relative to the decimal point. Think about these questions as you look at it:

- What do you notice about the fraction names of each place value as you move to the right from the decimal point?

- Why are these names useful in writing fractions as decimals?
Do the following Investigations on your white board with your partner and explain in writing how you know. Record your final answers here after our do the work with your partner.

Use your knowledge of fraction benchmarks and decimal place value to identify the greater number in each pair below. Use the greater than (>), less than (<), or equal to (=) symbols in writing your answers.

1. 0.1 and 0.9

2. 0.3 and 0.33

3. 0.25 and 0.250

4. 0.12 and 0.125

5. -0.1 and 0.1

6. -0.3 and -0.27
3.4 Day 2 Investigation

Each number line below has two points labeled with decimal numbers and one with a question mark. In each case, what decimal number should go in place of the question mark?

1. 

2. 

3. 

4. 

5. 

0.499 ? 0.501
Write a decimal value that is between the two given numbers. If you can’t do this explain why not.

1. 0.1 and 0.9
2. 0.3 and 0.33
3. 0.25 and 0.250
4. 0.12 and 0.125
5. -0.1 and 0.1
6. -0.3 and -0.27

3.4 Summary

Focus Question: How do we use what we know about fractions to estimate and compare decimals?
3.5 Investigation

Earthquake Relief: Moving From Fractions to Decimals

On January 12, 2010, a 7.0-magnitude earthquake struck the country of Haiti. It destroyed many homes and caused major damage. Many people had no place to live and little clothing and food. In response, people from all over the world collected clothing, household items, and food to send to the victims of the earthquake.

Students at a middle school decided to collect food to distribute to families whose homes were destroyed. They packed what they collected into boxes to send to the families. The students had to solve some problems as they packed the boxes.

As you work on this problem, ask yourself

When is decimal or fraction notation more useful, and why?
Each grade was assigned different numbers of families for which to pack boxes. Each grade shared the supplies equally among the families they were assigned. They had bags and plastic containers to repack items for the individual boxes. They also had a digital scale that measured in kilograms (kg) and grams (g).

A 1. The sixth graders are packing six boxes. How much of each item should the students include in each box? Write your answer as a fraction and as a decimal.

Wheat Crackers:

Oranges:

Powdered Milk:
The seventh graders are packing ten boxes. How much of each item should the students include in each box? Write your answer as a fraction and as a decimal.

Cheddar Cheese:

Peanut Butter:

Apples:
The eighth graders are packing 14 boxes. How much of each item should the students include in each box? Write your answer as a fraction and as a decimal.

Saltines:  
Raisins:  

Oranges:  
Peanut Butter:  

Powdered Milk:  
Swiss Cheese:  

### 3.5 Summary

**Focus Question:** Why does it make sense to divide the numerator of a fraction by the denominator to find an equivalent decimal representation?
4.1 Investigation

Who is the Best? Making Sense of Percents

Sports statistics are often given in percents. An important statistic for basketball teams is the successful free-throw percent. You will use mathematics to compare the basketball statistics of two well-known men’s basketball teams in the NCAA.

How are free-throw shooting averages determined for basketball teams?

What is a percent?

Percent means ______________________

During a recent year, two NCAA basketball teams made 108 out of 126 free-throw attempts and 195 out of 257 attempts. It is difficult to tell which team was better at free throws that year using raw numbers. Therefore, sports announcers often give percents instead of raw numbers.

A. Will drew pictures similar to the fund-raising thermometers to help him think about the percent of free throws made by the two teams. Then he got stuck!

Help Will use the pictures he drew to decide which player is better at free throws.
1. For each bar, estimate the number of free throws that should go with each marked percent in the picture.

2. For each team, shade the percent bar to show how many free throws the team made and estimate that percentage.

B. Alisha said that she could get better estimates of each team’s free-throw percentage using the percent bars below. Copy and complete her percent bars to estimate each player’s free-throw percentage. Compare your answers for Question A.

How do you write a fraction as a percent?

How do you write a decimal as a percent?
C. Denzel Valentine was a basketball player for Michigan State. He shot lots of free throws. For the 2015-2016 season, he shot 95 free throws and made 81 of them.

What fraction did he make?

What decimal did he make?

What percent did he make?

D. Zak Irvin was a U of M basketball player. For the 2015-2016 season, he made 50 out of 76 free throws.

What fraction did he make?

What decimal did he make?

What percent did he make?

Who was the better free throw shooter?
After thinking about free-throw percentages. Will said that percents are like fractions. Alisha disagreed and said that percents are more like ratios. Do you agree more with Will or with Alisha? Explain.

4.1 Summary

Focus Question: How is a percent bar useful in making comparisons with decimals?
4.2 Investigation

Genetic Traits: Finding Percents

Have you ever heard of genes? (Not the “jeans” you wear, even though they sound the same.) What color are your eyes? Is your hair curly? Are your earlobes attached? You are born with a unique set of genes that help to determine these traits.

Scientists who study human traits such as eye and hair color are geneticists. Geneticists are interested in how common certain human traits are.

Look at the earlobe of a classmate. Is it attached or detached? The type of earlobe you have is a trait determined partly by your genes. Here is a description of four genetic traits:

- A widow’s peak is a V-shaped hairline.
- A dimple is a small indentation, usually near the mouth.
- Straight hair does not have natural waves or curls.
- An earlobe is attached if its lowest point is attached directly to the head.
1. Copy and complete the table of genetic traits below.

<table>
<thead>
<tr>
<th>Trait</th>
<th>Yes</th>
<th>No</th>
<th>Total</th>
</tr>
</thead>
<tbody>
<tr>
<td>Attached Earlobes</td>
<td>12</td>
<td></td>
<td>30</td>
</tr>
<tr>
<td>Dimples</td>
<td>7</td>
<td></td>
<td>30</td>
</tr>
<tr>
<td>Straight Hair</td>
<td>24</td>
<td></td>
<td>30</td>
</tr>
<tr>
<td>Widow's Peak</td>
<td>17</td>
<td></td>
<td>30</td>
</tr>
</tbody>
</table>

2. For each trait, use a percent bar or another strategy to estimate the percent of people in the class who have that trait.

3. Using the percents from part (2) as rates, how many people in a school of 500 are likely to have a widow’s peak?

B Marjorie wanted to find the percent of students in her class with dimples. She said that she could get a very good estimate of the percent of students with any trait by using a bar with a mark for 1% like the one below.

1. How many students are in Marjorie’s class?

2. How did Marjorie figure out that 3.4 is at the 10% mark and 13.6 is at the 40% mark?
3. How many students in Marjorie’s class have dimples?
4. About what percent of students in Marjorie’s class have dimples?

5. How do you think Marjorie found that percent?

6. Are dimples more common in Marjorie’s class than in your class? Explain.

How is using a percent bar like using a rate table?
4.2 Summary

**Focus Question:** How can partitioning be used to express one number as a percent of another number?
4.3 Investigation

The Art of Comparison: Using Ratios and Percents

Do you have a favorite work of art? Is it by a famous artist such as Claude Monet, Georgia O’Keefe, or is it by your little sister?

Art museums own more pieces than they can display at one time. This means that art must be stored when it is not hanging in a gallery. A museum curator chooses which works to exhibit.

The Walker Art Center in Minneapolis, Minnesota held an exhibit entitled $\frac{50}{50}$. For the exhibit, the public voted via the Internet on which pieces they wanted the museum to display, and curators chose the remaining pieces.

- What do you think $\frac{50}{50}$ refers to in the title?

Another art museum held a similar $\frac{50}{50}$ exhibit. The picture shows the public’s part of the exhibit. The curators’ part is covered up.

How many works of art do you estimate were in the exhibit? Is there any other information that would help you make a better estimate? Explain.
B  Below is a picture of the complete exhibit. How does this picture change your estimate from Question A?

C  The picture in Question B shows you about $\frac{2}{3}$ of each part of the exhibit.
   1. Make a drawing to show the size of the whole exhibit.
   2. Use your drawing to estimate the number of works in each part.

D  1. Estimate the percent of the exhibit chosen by the public. Estimate the percent chosen by curators.
2. Use the percents from part (1). If there were 200 pieces in the exhibit, how many artworks do you think the public chose? How many do you think the curators chose?

What title would you choose for this exhibit using percents and ratios?

4.3 Summary

Focus Question: In what way is a percent like a ratio and like a fraction?